DHRUVA ACADEMY

ICSE | CBSE | STATE
(8th | 9th | 10th)

MATHEMATICS

Content	Page Number
ICSE Question Paper 2008 & Solutions	Pg 1
ICSE Question Paper 2009 & Solutions	Pg 15
ICSE Question Paper 2010 & Solutions	Pg 30
ICSE Question Paper 2011 & Solutions	Pg 47
ICSE Question Paper 2012 & Solutions	Pg 62
ICSE Question Paper 2013 & Solutions	Pg 77
ICSE Question Paper 2014	Pg 96
ICSE Question Paper 2015 & Solutions	Pg 103
ICSE Question Paper 2016 & Solutions	Pg 127
ICSE Question Paper 2018 & Solutions	Pg 155
ICSE Question Paper 2019 & Solutions	Pg 183
ICSE Question Paper 2020 & Solutions	Pg 213
ICSE Question Paper 2021 & Solutions	Pg 225

ICSE Question Paper (2008)

MATHEMATICS

SECTION A [40 MARKS]

*(Answer **all** questions from this Section.)*

Question 1.

(a) *The simple interest on a sum of money for 2 years at 4% per annum is ₹ 340. Find :*

(i) *the sum of money and*

(ii) *the compound interest on this sum for one year payable half yearly at the same rate.* **[3]**

(b) *If* $\frac{8a-5b}{8c-5d} = \frac{8a+5b}{8c+5d}$, *prove that* $\frac{a}{b} = \frac{c}{d}$ **[3]**

(c) *If* $(x-2)$ *is a factor of* $2x^3 - x^2 - px - 2$

(i) *find the value of p.*

(ii) *with the value of p, factorize the above expression completely.* **[4]**

Solution.

(a) (i) *Given* : S.I. = ₹ 340, T = 2Years, R = 4%, P = ?

$$\text{S.I.} = \frac{\text{P} \times \text{R} \times \text{T}}{100}$$

or
$$\text{P} = \frac{\text{S.I.} \times 100}{\text{R} \times \text{T}} = \frac{340 \times 100}{4 \times 2}$$

$$= ₹\ 4250$$ **Ans.**

(ii) *Given* : T = 1 Years = $1 \times 2 = 2$ times, R = 4%, $\Rightarrow$ R = 2% (for half yearly.)

$$\text{C.I.} = \text{P}\left[\left(1 + \frac{\text{R}}{100}\right)^{\text{T}} - 1\right]$$

$$= 4250\left[\left(1 + \frac{2}{100}\right)^{2} - 1\right]$$

$$= 4250\left[\frac{101}{2500}\right] = ₹\ 171{\cdot}70$$ **Ans.**

(b) *Given* :

$$\frac{8a+5b}{8c+5d} = \frac{8a-5b}{8c-5d}$$

$$\frac{8a+5d}{8a-5b} = \frac{8c+5d}{8c-5d}$$ (Apply alternendo)

$$\frac{8a+5b+8a-5b}{8a+5b-8a+5b} = \frac{8c+5d+8c-5d}{8c+5d-8c+5d}$$

(Apply componendo and dividendo rule)

$$\frac{16a}{10b} = \frac{16c}{10d}$$

$$\frac{a}{b} = \frac{c}{d}$$ **Proved**

(c) $(x-2)$ is a factor of $2x^3 - x^2 - px - 2$

(i) $\Rightarrow x = 2$ will satisfy this equation.

$\Rightarrow 2 \cdot 2^3 - 2^2 - 2p - 2 = 0$

$\Rightarrow 16 - 4 - 2p - 2 = 0$

$10 - 2p = 0$

$\Rightarrow 2p = 10$

$\Rightarrow p = 5$ **Ans.**

$$\begin{array}{r|l} & 2x^2 + 3x + 1 \\ \hline x-2 & 2x^3 - x^2 - 5x - 2 \\ & \underline{2x^3 - 4x^2} \quad - \quad + \\ & 3x^2 - 5x - 2 \\ & \underline{3x^2 - 6x} \quad - \quad + \\ & x - 2 \\ & \underline{x - 2} \quad - \quad + \\ & \times \end{array}$$

(ii) On dividing $2x^3 - x^2 - 5x - 2$ by $x - 2$, we get

$\Rightarrow (x-2)(2x^2 + 3x + 1)$

$\Rightarrow (x-2)(2x^2 + 2x + x + 1)$

$\Rightarrow (x-2)(2x(x+1) + 1(x+1))$

$\Rightarrow (x-2)(x+1)(2x+1)$ **Ans.**

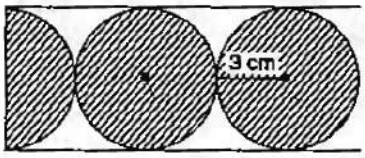
Question 2.

(a) *Solve the given inequation and graph the solution on the number line.*

$$2y - 3 < y + 1 \leq 4y + 7;\ y \in R.$$ **[3]**

(b) *In the given figure, find the area of the unshaded portion within the rectangle.*

(Take $\pi = 3{\cdot}14$) **[3]**



(c) *A shopkeeper buys a camera at a discount of 20% from the wholesaler, the printed price of the camera being ₹ 1600 and the rate of sales tax is 6%. The shopkeeper sells it to the buyer at the printed price and charges tax at the same rate. Find :*

(i) *The price at which the camera can be bought.*

(ii) *The VAT (Value Added Tax) paid by the shopkeeper.* **[4]**

Solution.

(a) *Given :* $2y - 3 < y + 1 \leq 4y + 7, y \in R$

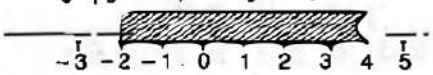
$2y - 3 < y + 1$ | $y + 1 \leq 4y + 7$

$2y - y < 3 + 1$ | $4y - y \geq 1 - 7$

$\Rightarrow y < 4$ | $3y \geq -6$

$y \geq -2$

solution set : $\{y \mid y \in R, -2 \leq y < 4\}$



(b) Length of rectangle = 15 cm

Breadth of rectangle = 6 cm.

Area of rectangle = 15×6

$= 90\ cm^2$

Area of circle $= \pi r^2$
$= 3{\cdot}14 \times 9$
$= 28{\cdot}26 \text{ cm}^2.$

Area of shaded portion = Area of $2\frac{1}{2}$ circle
$= 28{\cdot}26 + 28{\cdot}26 + 14{\cdot}13$
$= 70{\cdot}65 \text{ cm}^2.$

Area of unshaded portion in the rectangle = Area of the rectangle − Area of shaded portion
$= 90 - 70{\cdot}65$
$= 19{\cdot}35 \text{ cm}^2$ **Ans.**

(c). (i) Cost of camera for buyer = Printed Price + Sales tax on it
$= 1600 + \frac{6}{100} \times 1600$
$= ₹\,1696$ **Ans.**

(ii) Discount on printed price $= \frac{20}{100} \times 1600 = ₹\,320.$
Cost Price of the camera $= 1600 - 320 = ₹\,1280.$
Sales tax $= \frac{6}{100} \times 1280 = ₹\,76{\cdot}80$
Tax paid by shopkeeper $= \frac{6}{100} \times 1600 = ₹\,96.$
VAT paid by shopkeeper $= 96 - 76{\cdot}80$
$= ₹\,19{\cdot}20$ **Ans.**

Question 3.

(a) *David opened a Recurring Deposit Account in a bank and deposited ₹ 300 per month for two years. If he received ₹ 7725 at the time of maturity, find the rate of interest per annum.* **[3]**

(b) *If* $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$, *find the Matrix M.* **[3]**

(c) *Use a graph paper for this question. (Take 1 cm = 1 unit on both the axes). Plot the points A (–2, 0), B (4, 0), C (1, 4) and D (–2, 4).*

(i) *Draw the line of symmetry of Δ ABC. Name it* L_1.

(ii) *Point D is reflected about the Line* L_1 *to get the image E. Write the coordinates of E.*

(iii) *Name the figure ABED.*

(iv) *Draw all the lines of symmetry of the figure ABED.* **[4]**

Solution.

(a) *Given* : Deposited per month (P) = ₹ 300, n = 2 Year = 24 months, Amount = ₹ 7725, R = ?

Principal equivalent to 1 month $= P \times \frac{n(n+1)}{2} = \frac{300 \times 24 \times 25}{2}$
$= 300 \times 12 \times 25 = ₹\,90{,}000$
$A = P + SI$
$\Rightarrow \quad 7725 = 24 \times 300 + \frac{90000 \times R \times 1}{12 \times 100}$

$\Rightarrow \quad 525 = \frac{900 \times R}{12}$

$\Rightarrow \quad R = \frac{525 \times 12}{900} = 7\%.$ **Ans.**

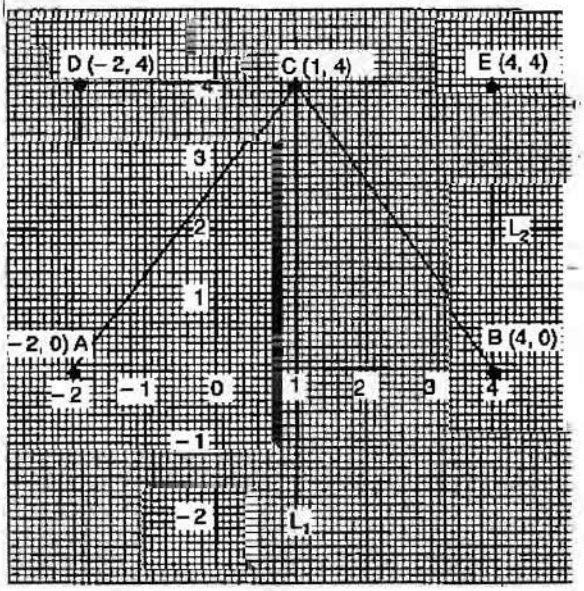
(b) *Given* : $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$

$\Rightarrow \quad 2M = \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$

$\Rightarrow \quad = \begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix}$

$M = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix}$ **Ans.**

(c) (i)



(ii) Co-ordinate of E (4, 4)

(iii) ABED is rectangle.

(iv) See figure (L_1 and L_2 are the line of symmetry of ractangle ABED)

Question 4.

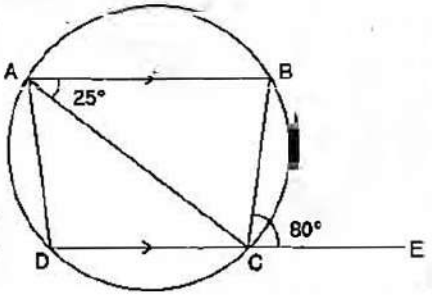
(a) *Without using tables, evaluate :* $\frac{\sin 25°}{\sec 65°} + \frac{\cos 25°}{\text{cosec } 65°}$ **[3]**

(b) *In the alongside figure, AB is parallel to DC,* $\angle BCE = 80°$ *and* $\angle BAC = 25°$. *Find :*

(i) $\angle$ CAD,

(ii) $\angle$ CBD,

(iii) $\angle$ ADC. **[3]**



(c) *Mr. Dhoni has an account in the Union Bank of India. The following entries are from his pass book :*

Date	Particulars	Withdrawals (in ₹)	Deposits (in ₹)	Balance (in ₹)
Jan 3, 07	B/F	—	—	2642.00
Jan 16	To Self	640.00	—	2002.00
March 5	By Cash	—	850.00	2852.00
April 10	To Self	1130.00	—	1722.00
April 25	By Cheque	—	650.00	2372.00
June 15	By Cash	577.00	—	1795.00

Calculate the interest from January 2007 to June 2007 at the rate of 4% per annum. **[4]**

Solution.

(a) *Given :* $\dfrac{\sin 25^\circ}{\sec 65^\circ} + \dfrac{\cos 25^\circ}{\operatorname{cosec} 65^\circ}$

$= \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$= \sin 25^\circ \cos (90^\circ - 25^\circ) + \cos 25^\circ \sin (90^\circ - 25^\circ)$

$= \sin 25^\circ \sin 25^\circ + \cos 25^\circ \cos 25^\circ$

$= \sin^2 25^\circ + \cos^2 25^\circ = 1$ ($\because \sin^2\theta + \cos^2\theta = 1$) **Ans.**

(b) (i) $\angle BCE = \angle BAD = 80^\circ$

($\because$ ext. of cyclic quad. is equal to opp. int. angle.)

$\angle CAD = \angle BAD - \angle BAC = 80^\circ - 25^\circ = 55^\circ$ **Ans.**

(ii) $\angle CBD = \angle CAD$ ($\because$ Angle of the same segment are equal.)

$= 55^\circ$ **Ans.**

(iii) $AB \parallel DC$ (given)

$\angle BAD + \angle ADC = 180^\circ$

$\Rightarrow (25 + 55) + \angle ADC = 180^\circ$

$= 180 - 80 = 100^\circ$ ($\because$ ABCD is cyclic quad.)

Ans.

(c)

Minimum Balance for January	= ₹	2,002
Minimum Balance for February	= ₹	2,002
Minimum Balance for March	= ₹	2,852
Minimum Balance for April	= ₹	1,722
Minimum Balance for May	= ₹	2,372
Minimum Balance for June	= ₹	1,795
Total	= ₹	12,745

$\text{S.I.} = \dfrac{P \times R \times T}{100}$

$= \dfrac{12745 \times 4 \times 1}{100 \times 12}$

$= ₹\,42{\cdot}48$ **Ans.**

SECTION B [40 Marks]

Answer any Four Questions in this Section.

Question 5.

(a) *A function in x is defined as :* *

$$f(x) = \frac{x+2}{2x-1}; x \in R \text{ and } x \neq \frac{1}{2},$$

Find : (i) $f(-3)$,

(ii) $f(x-1)$,

(iii) x *if* $f(x) = 1$. **[3]**

(b) *Prove the identity :* $\frac{\sin A}{1+\cos A} = \text{cosec } A - \cot A$. **[3]**

(c) *If A = (–4, 3) and B = (8, –6)*

(i) *Find the length of AB.*

(ii) *In what ratio is the line joining AB, divided by the x–axis ?* **[4]**

Solution.

(b)

$$\text{L.H.S.} = \frac{\sin A}{1+\cos A}$$

$$= \frac{\sin A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}$$

$$= \frac{\sin A(1-\cos A)}{1-\cos^2 A}$$

$$= \frac{\sin A(1-\cos A)}{\sin^2 A}$$

$$\Rightarrow = \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$

$$= \text{cosec } A - \cot A = \text{R.H.S.}$$ **Proved**

(c) (i) *Given :* A = (– 4, 3), B = (8, – 6)

$$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(8+4)^2 + (-6-3)^2}$$

$$= \sqrt{144+81}$$

$$= \sqrt{225} = 15.$$ **Ans.**

(ii) Let any point on x–axis will be P $(x, 0)$.

$$x = \frac{mx_1 + nx_2}{m+n},$$

$$y = \frac{my_1 + ny_2}{m+n}$$

$$0 = \frac{m \cdot 3 + n \cdot (-6)}{m+n}$$

$$\Rightarrow 3m = 6n$$

$$\Rightarrow \frac{m}{n} = \frac{6}{3} = \frac{2}{1}$$

The ratio will be 2 : 1. **Ans.**

Question 6.

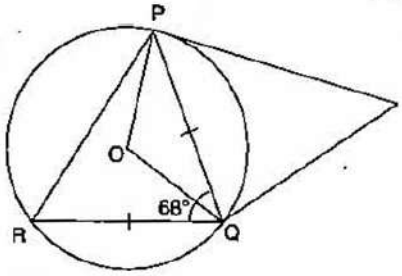
(a) *Solve the following quadratic equation for x and give your answer correct to two decimal places :*

$$5x\,(x+2) = 3$$ **[3]**

(b) *In the figure given alongside PQ = QR, ∠ RQP = 68°, PC and CQ are tangents to the circle with centre O. Calculate the values of :*

(i) $\angle QOP$,

(ii) $\angle QCP$. **[3]**



(c) *A company with 4000 shares of nominal value of ₹ 110 each declares an annual dividend of 15%. Calculate :*

(i) *The total amount of dividend paid by the company.*

(ii) *The annual income of Shah Rukh who holds 88 shares in the company.*

(iii) *If he received only 10% on his investment, find the price Shah Rukh paid for each share.* **[4]**

Solution.

(a) *Given :* $5x(x+2) = 3$

$$5x^2 + 10x - 3 = 0$$

Camparing with $ax^2 + bx + c = 0$, we get $a = 5, b = 10$ and $c = -3$.

We know that $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{(10)^2 - 4 \times 5 \times (-3)}}{2 \times 5}$$

$$= \frac{-10 \pm \sqrt{100 + 60}}{10}$$

$$= \frac{-10 \pm 4\sqrt{10}}{10} = \frac{-5 \pm 2\sqrt{10}}{5}$$

$$= \frac{-5 \pm 2 \times 3{\cdot}16}{5} = \frac{-5 \pm 6{\cdot}32}{5}$$

Hence $x = 0{\cdot}26$ or $-2{\cdot}26$. **Ans.**

(b) *Given :* PQ = QR, ∠ RQP = 68°

(i) In Δ PQR, $PQ = RQ$

$\therefore \quad \angle PRQ = \angle QPR$

$\Rightarrow \quad \angle PRQ + \angle QPR + 68° = 180°$ [sum of the angle of a Δ is 180°]

$\Rightarrow \quad 2\angle PRQ = 180 - 68 = 112°$

$\Rightarrow \quad \angle PRQ = 56°$

$\therefore \quad \angle QOP = 2 \times \angle PRQ$

[angle at centre of the circle is twice the angle of at the remaining circumference]

$= 2 \times 56 = 112°$ **Ans.**

(ii) $\angle PQC = \angle PRQ$

$\angle QPC = \angle PRQ$

$\Rightarrow \angle PQC = \angle QPC = 56°$

$\Rightarrow \angle PQC + \angle QPC + \angle PCQ = 180°$

$\Rightarrow 56° + 56° + \angle PCQ = 180°$

$\Rightarrow \angle QCP = 180° - 56° - 56°$

$= 68°$ **Ans.**

(c) *Given* : No. of shares = 4000, NV = ₹ 110, Dividend = 15%

(i) Dividend on 1 share $= \frac{15}{100} \times 110$

$= ₹ 16{\cdot}50$

Total dividend $= 4000 \times 16{\cdot}50$

$= ₹ 66000$ **Ans.**

(ii) Income on 88 shares $= 88 \times 16{\cdot}50$

$= ₹ 1{,}452$

Annual income of Shah Rukh = ₹ 1,452. **Ans.**

(iii) Let his investment $= x$

$\frac{10}{100} \times x = ₹ 1452$

$x = ₹ 14520$

$\therefore$ Price for each share $= \frac{14520}{88} = ₹ 165$. **Ans.**

Question 7.

(a) *The income of Mr. Bachhan was as follows :* **

- *Basic Salary* : *₹ 20,000 per month*
- *Dearness Allowance* : *₹ 12,000 per month*
- *Interest from Bank* : *₹ 16,000 for the whole year.*

Savings :

- *Contribution towards Provident Fund* : *15% of Basic salary*
- *National Savings Certificate* : *₹ 40,000*
- *Contribution towards LIC premium* : *₹ 30,000 per year*

Donations

- *To National Defence Fund* : *₹ 12,000 (eligible for 100% exemption)*

If a sum of ₹ 3,000 was deducted every month towards Income tax from his salary for the first 11 months of the year, calculate the tax Mr. Bachhan has to pay in the last month of the financial year : **[6]**

Tax slab :

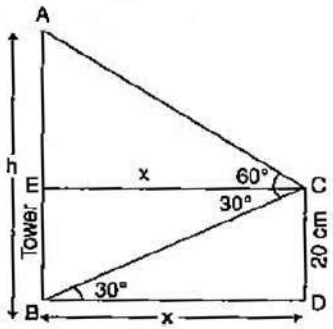
Upto ₹ 1,00,000	:	*No tax.*
From ₹ 1,00,001 to ₹ 1,50,000	:	*10% of the income exceeding ₹ 1,00,000*
From ₹ 1,50,001 to ₹ 2,50,000	:	*₹ 5,000+ 20% of the income exceeding ₹ 1,50,000*
Above ₹ 2,50,000	:	*₹ 25,000 + 30% of the income exceeding ₹ 2,50,000*
Deductions against savings	:	*Upto a maximum amount of ₹ 1,00,000*
Education Cess	:	*2% of the tax payable.*

** *Solution has not given due to out of present syllabus.*

(b) *A vertical pole and a vertical tower are on the same level ground. From the top of the pole the angle of elevation of the top of the tower is 60° and the angle of depression of the foot of the tower is 30°. Find the height of the tower if the height of the pole is 20 m.* **[4]**

Solution.

(b) Let h be the height of tower and x be the distance between tower and pole.



In rt. Δ BCE, $\tan 30° = \frac{20}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{20}{x}$

$\Rightarrow x = 20\sqrt{3}$ m

In rt. Δ ECA, $\tan 60° = \frac{h-20}{x}$

$\Rightarrow \frac{\sqrt{3}}{1} = \frac{h-20}{x}$

$\Rightarrow h - 20 = x\sqrt{3}$

$\Rightarrow h - 20 = 20\sqrt{3} \times \sqrt{3} = 60$ m

$\Rightarrow h = 80$ m. **Ans.**

Question 8.

(a) *Find the H.C.F. of the given polynomials :* **

$$x^2 - \frac{1}{a^2} \text{ and } x^2 + \frac{2x}{a} + \frac{1}{a^2}$$ **[3]**

(b) *Using a ruler and a pair of compasses only, construct :*

(i) *A triangle ABC, given AB = 4 cm, BC = 6 cm and $\angle ABC = 90°$.*

(ii) *A circle which passes through the points A, B and C and mark its centre as O.* **[3]**

(c) *Points A and B have coordinates (7, –3) and (1, 9) respectively. Find :*

(i) *The slope of AB.*

(ii) *The equation of the perpendicular bisector of the line segment AB.*

(iii) *The value of 'p' if (–2, p) lies on it.* **[4]**

Solution.

(b) Steps of Construction :

(1) Draw side BAC, 6 cm.

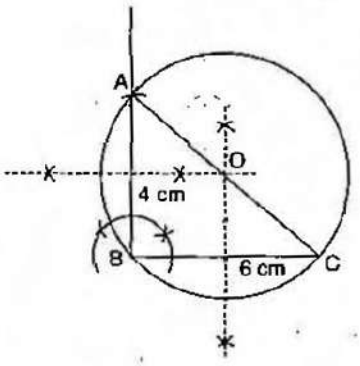
(2) Draw a $\angle B = 90°$, and cut AB = 4 cm.

(3) Meet AC.

(4) Draw Bisector of BC and AB. Which meet at point 'O'.

(5) Now draw a circle as centre 'O'.

(6) This circle passes through the point A, B, and 'C'.

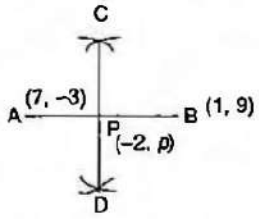


** *Solution has not given due to out of present syllabus.*

(c) (i) Slope of AB $(m_1) = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{9+3}{1-7} = \frac{12}{-6} = -2$

Ans.



(ii) The mid point of line AB $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{7+1}{2}, \frac{-3+9}{2}\right) = (4, 3)$

Slope of perpendicular bisector of AB

$$m_2 = \frac{-1}{m_1} = \frac{-1}{-2} = \frac{1}{2}$$

Equation of perpendicular bisector

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - 3 = \frac{1}{2}(x - 4)$$

$$\Rightarrow \quad 2y - 6 = x - 4$$

$$\Rightarrow \quad x - 2y + 2 = 0 \qquad \textbf{Ans.}$$

(iii) Given point $(-2, p)$ lies on the equation $x - 2y + 2 = 0$

$$-2 - 2p + 2 = 0$$

$$\Rightarrow \quad -2p = 0$$

$$\Rightarrow \quad p = 0 \qquad \textbf{Ans.}$$

Question 9.

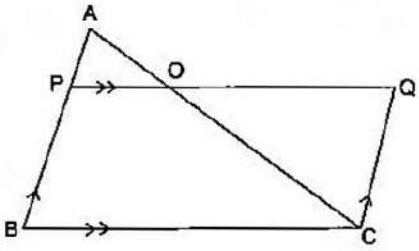
(a) *Given* $A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$ *and* $BA = C^2$.

Find the values of p and q. **[3]**

(b) *In* ΔABC, $AP : PB = 2 : 3$. *PO is parallel to BC and is extended to Q so that CQ is parallel to BA. Find :*

(i) *Area* ΔAPO *: area* ΔABC.

(ii) *Area* ΔAPO *: area* ΔCQO. **[3]**



(c) *The volume of a conical tent is 1232* m^3 *and the area of the bare floor is 154* m^2. *Calculate the :*

(i) *Radius of the floor.*

(ii) *Height of the tent.*

(iii) *Length of the canvas required to cover this conical tent if its width is 2 m.*

[4]

Solution.

(a) *Given* : $A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$

$$BA = C^2$$

$$\Rightarrow \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0\times p + (-q)\times 0 & 0\times 0 + (-q)\times 2 \\ 1\times p + 0\times 0 & 1\times 0 + 0\times 2 \end{bmatrix} = \begin{bmatrix} 2\times 2 + (-2)\times 2 & 2\times(-2) + (-2)\times 2 \\ 2\times 2 + 2\times 2 & 2\times(-2) + 2\times 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$$

$$\Rightarrow p = 8, q = 4 \quad \textbf{Ans.}$$

(b) (i) $$\frac{AP}{PB} = \frac{2}{3} = \frac{AO}{OC}$$

$$\frac{AP}{AP + PB} = \frac{2}{2+3}$$

$$\Rightarrow \frac{AP}{AB} = \frac{2}{5} \quad \ldots(1)$$

PO is parallel to BC and CQ is parallel to BA.
So, PBCQ is a parallelogram.

$$\Rightarrow PB = CQ$$

$$\Rightarrow \frac{AP}{PB} = \frac{2}{3} = \frac{AP}{CQ}$$

In Δ APO and Δ ABC,

$$\angle APO = \angle ABC \quad (\because PO \parallel BC)$$

$$\angle A = \angle A$$

$$\Rightarrow \Delta APO \sim \Delta ABC$$

$$\frac{AP}{AB} = \frac{AO}{AC} = \frac{2}{5}$$

$$\Rightarrow \frac{\text{area of } \Delta APO}{\text{area of } \Delta ABC} = \frac{AP^2}{AB^2}$$

[Ratio between the areas of two similar Δ's is equal to the ratio between the squares of their corresponding sides]

$$= \left(\frac{2}{5}\right)^2 = \frac{4}{25} \quad \textbf{Ans.}$$

(ii) In Δ APO and Δ CQO, $\angle AOP = \angle COQ$ (vertically opp.)

$$\angle OAP = \angle OCQ$$

$$\Rightarrow \Delta AOP \sim \Delta COQ \quad \text{(By A.A axiom)}$$

So $$\frac{\text{area of } \Delta APO}{\text{area of } \Delta CQO} = \frac{AP^2}{CQ^2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9} \quad \textbf{Ans.}$$

(c) *Given* : Volume (V) = 1232 m, Area of base = 154 m^2
Let r be the radius and h be the height.

(i) $$\text{Area} = \pi r^2 = 154$$

$$\frac{22}{7} r^2 = 154$$

$$r^2 = 49 \quad \Rightarrow \quad r = 7 \text{ m} \quad \textbf{Ans.}$$

(ii) $$\text{Volume (V)} = \frac{1}{3}\pi r^2 h = 1232$$

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times h = 1232$$

$$h = \frac{1232 \times 3}{22 \times 7} = 24 \text{ m.}$$ **Ans.**

(iii) Let l be the slant height of the conical tent, then

$$l = \sqrt{h^2 + r^2} = \sqrt{24^2 + 7^2} = 25\text{m}$$

Area of canvas required = Curved surface area of the tent

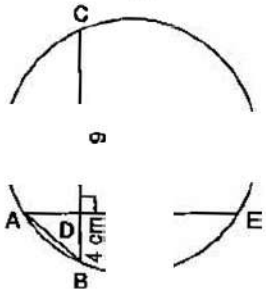
$\Rightarrow$ Length × width of canvas = πrl (for tent)

$\Rightarrow$ Length of canvas × 2 = $\frac{22}{7} \times 7 \times 25$

$$\text{Length of canvas} = \frac{550}{2} = 275 \text{ m.}$$ **Ans.**

Question 10.

(a) *In the given figure, AE and BC intersect each other at point D. If ∠ CDE = 90°, AB = 5 cm, BD = 4 cm and CD = 9 cm, find DE.* **[3]**



(b) *A straight line AB is 8 cm long. Locate by construction the locus of a point which is :*

(i) *Equidistant from A and B.*

(ii) *Always 4 cm from the line AB.*

(iii) *Mark two points X and Y, which are 4 cm from AB and equidistant from A and B, Name the figure AXBY.* **[3]**

(c) *Some students planned a picnic. The budget for the food was ₹ 480. As eight of them failed to join the party, the cost of the food for each member increased by ₹ 10. Find how many students went for the picnic.* **[4]**

Solution.

(a) *Given* : ∠ CDE = 90°, AB = 5 cm, BD = 4 cm, CD = 9 cm, DE = ?

$$AD = \sqrt{AB^2 - DB^2} = \sqrt{25 - 16} = 3$$

DA × DE = DB × DC (Product of the length of their segment is equal.)

$\Rightarrow$ $3 \times DE = 4 \times 9$

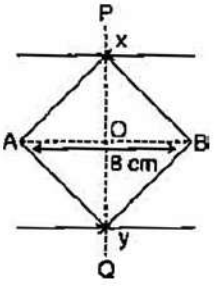
$\Rightarrow$ DE = 12 cm **Ans.**

(b) Steps of Construction :

(i) Draw AB is 8 cm, Draw PQ the perpen-dicular bisector of AB.

(ii) Draw CD and EF both parallel to AB and each at a distance of 4 cm from AB.

(iii) AXBY is a square.



(c) Let the no. of students planned a picnic = x.

$$\text{Budget for food} = ₹480$$

$$\text{Budget for each student} = \frac{480}{x}$$

If eight student failed to join the party, the cost for each student $= \frac{480}{x-8}$

$$\frac{480}{x-8} - \frac{480}{x} = 10$$

$$\frac{480x - 480(x-8)}{x(x-8)} = 10$$

$$\Rightarrow \quad \frac{480x - 480x + 3840}{x(x-8)} = 10$$

$$\Rightarrow \quad x^2 - 8x - 384 = 0$$

$$\Rightarrow \quad x^2 - 24x + 16x - 384 = 0$$

$$x(x-24) + 16(x-24) = 0$$

$$\Rightarrow \quad (x-24)(x+16) = 0$$

$$\Rightarrow \quad x = 24, -16$$

But the number of students can not be negative.

$$\therefore \quad x = 24$$

$\therefore$ The number of student who went for picnic $= x - 8 = 24 - 8 = 16$ **Ans.**

Question 11.

(a) *The weight of 50 apples were recorded as given below. Calculate the mean weight, to the nearest gram, by the Step Deviation Method.* **[5]**

Weight in grams	*80–85*	*85–90*	*90–95*	*95–100*	*100–105*	*105–110*	*110–115*
No. of apples	*5*	*8*	*10*	*12*	*8*	*4*	*3*

(b) *Using a graph paper, draw an ogive for the following distribution which shows the marks obtained in the General Knowledge paper by 100 students.*

Marks	*0–10*	*10–20*	*20–30*	*30–40*	*40–50*	*50–60*	*60–70*	*70–80*
No. of students	*5*	*10*	*20*	*25*	*15*	*12*	*9*	*4*

Use the ogive to estimate :

(i) *The median.*

(ii) *The number of students who score marks above 65.* **[5]**

Solution.

(a)

Weight	*No. of apples* (f_i)	*Mid Value* (x)	$u_i = \frac{x-A}{C}$	f_iu_i
80–85	5	82·5	–3	–15
85–90	8	87·5	–2	–16
90–95	10	92·5	–1	–10
95–100	12	97·5	0	0
100–105	8	102·5	1	8
105–110	4	107·5	2	8
110–115	3	112·5	3	9
	$\Sigma f_i = 50$			$\Sigma f_iu_i = -16$

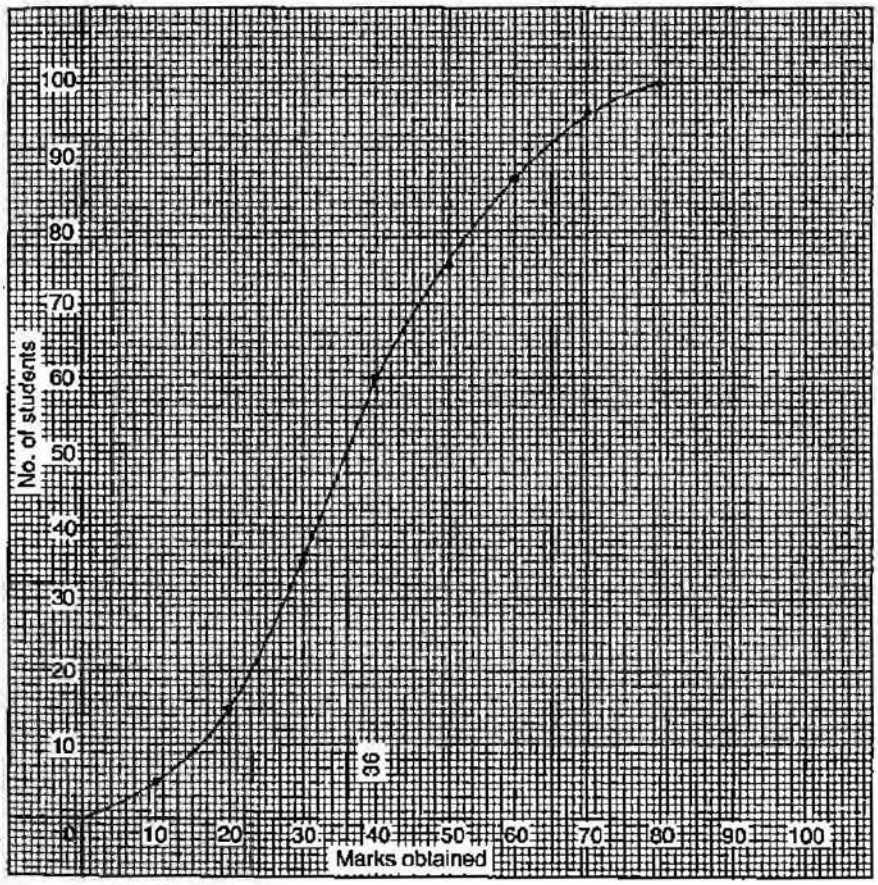
Taking assumed mean (A) = 97·5, C = 5

$$\text{Mean} = A + C \times \frac{\Sigma f_i u_i}{\Sigma f_i} = 97{\cdot}5 + 5 \times \frac{-16}{50}$$

$$= 97{\cdot}5 - 1{\cdot}6 = 95{\cdot}9 = 96$$ **Ans.**

(b)

Marks	*No. of students*	*c.f.*
0–10	5	5
10–20	10	15
20–30	20	35
30–40	25	60
40–50	15	75
50–60	12	87
60–70	9	96
70–80	4	100
	100	



Here $n = 100$ which is even.

(i) $$\text{Median} = \left(\frac{n}{2}\right)^{\text{th}} \text{term} = 50^{\text{th}} \text{ term}$$

From the graph, we get

The required median = 36

(ii) The number of student who score above 65 = 100 – 92 = 8. **Ans.**

ICSE Paper Question (2009)

MATHEMATICS

SECTION A [40 Marks]

*(Answer **all** questions from this Section.)*

Question 1.

(a) *Mr. Dubey borrows ₹ 1,00,000 from State Bank of India at 11% per annum compound interest. He repays ₹ 41,000 at the end of the first year and ₹ 47,700 at the end of the second year. Find the amount outstanding at the beginning of the third year.* **[3]**

(b) *A dice is thrown once. What is the probability that the*

(i) *number is even*

(ii) *number is greater than 2 ?* **[3]**

(c) *Find the HCF and LCM of the following polynomials :* ** **[4]**

$$3x^3 - 27x^2 + 60x \quad \text{and} \quad x^2 - 16$$

Solution :

(a) *Given* : P = ₹ 1,00,000, R = 11%

$$\text{Interest for first year} = \frac{PRT}{100} = \frac{1,00,000 \times 11 \times 1}{100} = ₹\,11,000$$

$$\text{Amount after first year} = 1,00,000 + 11,000 = ₹\,1,11,000$$

$$\text{Principal for second year} = ₹\,1,11,000 - ₹\,41,000$$

$$= ₹\,70,000$$

$$\text{Interest for second year} = \frac{70,000 \times 11 \times 1}{100} = ₹\,7,700$$

$$\text{Amount after second year} = ₹\,70,000 + 7,700 = 77,700$$

Amount outstanding for beginning of third year

$$= ₹\,77,700 - ₹\,47,700$$

$$= ₹\,30,000.$$ **Ans.**

(b) Dice is thrown once.

$$\text{Sample space} = \{1, 2, 3, 4, 5, 6\}$$

$$\therefore \quad n(S) = 6$$

(i) $$\text{Number is even} = \{2, 4, 6\}$$

$$\therefore \quad n(E) = 3$$

$$P(\text{Even number}) = \frac{n(E)}{n(S)} = \frac{3}{6} = 1/2$$ **Ans.**

(ii) $$\text{Number is greater than 2} = \{3, 4, 5, 6\}$$

$$n(E) = 4$$

$$P(> 2) = \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$ **Ans.**

** *Solution has not given due to out of present syllabus.*

Question 2.

(a) *Find x and y, if* $\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$ **[3]**

(b) *What least number must be added to each of the numbers 5, 11, 19 and 37 so that they are in proportion ?* **[3]**

(c) *Given that x + 2 and x + 3 are factors of* $2x^3 + ax^2 + 7x - b$. *Determine the values of a and b.* **[4]**

Solution :

(a) *Given* :

$$\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 6x + 2x \\ 3y + 6y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 8x \\ 9y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\left.\begin{aligned} 8x = 16 &\Rightarrow x = 2 \\ 9y = 9 &\Rightarrow y = 1 \end{aligned}\right] \quad \textbf{Ans.}$$

(b) Let the number x be added to each number.

$$5 + x : 11 + x = 19 + x : 37 + x$$

$$\frac{5 + x}{11 + x} = \frac{19 + x}{37 + x}$$

By componendo and dividendo,

$$\Rightarrow \quad \frac{5 + x + 11 + x}{5 + x - 11 - x} = \frac{19 + x + 37 + x}{19 + x - 37 - x}$$

$$\frac{16 + 2x}{-6} = \frac{56 + 2x}{-18}$$

$$\Rightarrow \quad 3(16 + 2x) = 56 + 2x$$

$$48 + 6x = 56 + 2x$$

$$\Rightarrow \quad 4x = 8$$

$$\Rightarrow \quad x = 2 \quad \textbf{Ans.}$$

(c) *Given* : $(x + 2)$ and $(x + 3)$ are the factors of $2x^3 + 9x^2 + 7x - b$.

$\because f(-2)$ and $f(-3)$ will be zero.

$$f(x) = 2x^3 + ax^2 + 7x - b$$

$$f(-2) = 2(-2)^3 + a(-2)^2 + 7(-2) - b = 0$$

$$-16 + 4a - 14 - b = 0$$

$$4a - b = 30 \quad \ldots(1)$$

$$f(-3) = 2(-3)^3 + a(-3)^2 + 7(-3) - b = 0$$

$$-54 + 9a - 21 - b = 0$$

$$9a - b = 75 \quad \ldots(2)$$

Solving (1) and (2), we get

$$a = 9, \; b = 6 \quad \textbf{Ans.}$$

Question 3.

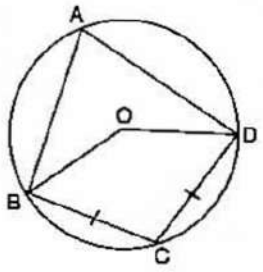
(a) *Solve the inequation and represent the solution set on the number line.*

$$-3 + x \le \frac{8x}{3} + 2 \le \frac{14}{3} + 2x, \text{ where } x \in I \qquad [3]$$

(b) *Find the value of p for which the lines*

$2x + 3y - 7 = 0$ *and* $4y - px - 12 = 0$ *are perpendicular to each other.* **[3]**

(c) *In the given figure O is the centre of the circle,* $\angle BAD = 75°$ *and chord BC = chord CD. Find : (i)* $\angle BOC$ *(ii)* $\angle OBD$ *(iii)* $\angle BCD$. **[4]**

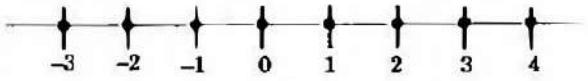


Solution :

(a) $$-3 + x \le \frac{8x}{3} + 2 \le \frac{14}{3} + 2x, x \in I$$

$-3 + x \le \frac{8x}{3} + 2$	$\frac{8x}{3} + 2 \le \frac{14}{3} + 2x$
$\frac{8x}{3} - \frac{x}{1} \ge -3 - 2$	$\frac{8x}{3} - 2x \le \frac{14}{3} - \frac{2}{1}$
$\frac{8x - 3x}{3} \ge -5$	$\frac{8x - 6x}{3} \le \frac{14 - 6}{3}$
$5x \ge -15$	$\frac{2x}{3} \le \frac{8}{3}$
$x \ge -3$	$x \le 4$

Solution set : $\{-3 \le x \le 4\}$



(b) Given equation is $2x + 3y - 7 = 0$

$\Rightarrow \quad 3y = -2x + 7$

$\Rightarrow \quad y = -\frac{2}{3}x + \frac{7}{3}$

Slope of the line $(m_1) = -\frac{2}{3}$

Another equation is $4y - px - 12 = 0$

$\Rightarrow \quad 4y = px + 12$

$\Rightarrow \quad y = \frac{p}{4}x + 3$

Slope of the line $(m_2) = \frac{p}{4}$

As per the question, lines are perpendicular.

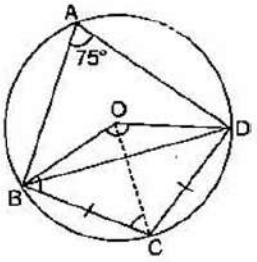
$$m_1 \times m_2 = -1$$

$$-\frac{2}{3} \times \frac{p}{4} = -1$$

$$\Rightarrow \quad -2p = -12$$

$$\Rightarrow \quad p = 6 \qquad \textbf{Ans.}$$

(c) *Given* : $\angle BAD = 75°$, chord BC = chord CD



$$\angle BOD = 2 \times \angle BAD = 2 \times 75° = 150°$$

(i) $$\angle BOC = \frac{1}{2} \angle BOD$$

$$= \frac{1}{2} \times 150° = 75° \qquad \textbf{Ans.}$$

(ii) $$\angle OBD = \frac{1}{2}(180° - 150°)$$

$$= 15° \qquad \textbf{Ans.}$$

(iii) $$\angle BCD = 180° - 75° = 105° \qquad \textbf{Ans.}$$

Question 4.

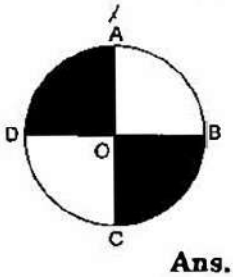
(a) *Find the mean, median and mode of the following distribution :*

8, 10, 7, 6, 10, 11, 6, 13, 10 **[3]**

(b) *Without using trigonometric tables evaluate the following :*

$$\frac{\sec 17°}{\text{cosec } 73°} + \frac{\tan 68°}{\cot 22°} + \cos^2 44° + \cos^2 46° \qquad \textbf{[3]}$$

(c) *AC and BD are two perpendicular diameters of a circle with centre O. If AC = 16 cm, calculate the area and perimeter of the shaded part. (Take $\pi = 3.14$)* **[4]**



Solution :

(a) $$\text{Mean} = \frac{\Sigma x}{n} = \frac{8 + 10 + 7 + 6 + 10 + 11 + 6 + 13 + 10}{9}$$

$$= \frac{81}{9} = 9 \qquad \textbf{Ans.}$$

For the median, we arrange the data in ascending order

$$6, 6, 7, 8, 10, 10, 10, 11, 13$$

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{term} = \left(\frac{9+1}{2}\right)^{\text{th}} \text{term}$$

$$= 5^{\text{th}} \text{ term} = 10$$

In the given data, 10 occurs maximum number of times, therefore

$$\text{Mode} = 10 \qquad \textbf{Ans.}$$

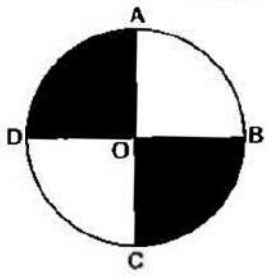
(b) *Given* : $$\frac{\sec 17°}{\text{cosec } 73°} + \frac{\tan 68°}{\cot 22°} + \cos^2 44° + \cos^2 46°$$

$$= \frac{\sec(90° - 73°)}{\text{cosec } 73°} + \frac{\tan(90° - 22°)}{\cot 22°} + \cos^2(90° - 46°) + \cos^2 46°$$

$$= \frac{\text{cosec } 73°}{\text{cosec } 73°} + \frac{\cot 22°}{\cot 22°} + \sin^2 46° + \cos^2 46°$$

$$= 1 + 1 + 1 = 3$$ **Ans.**

(c) *Given* : AC = 16 cm = diameter of the circle, $\pi = 3{\cdot}14$.



Area of shaded portion = 2 quadrant

$$= \frac{1}{2} \times \pi r^2$$

$$= \frac{1}{2} \times 3{\cdot}14 \times (8)^2$$

$$= 100{\cdot}48 \text{ cm}^2$$ **Ans.**

Perimeter of the shaded portion = πr + DB + AC

$$= 3{\cdot}14 \times 8 + 16 + 16$$

$$= 57{\cdot}12 \text{ cm.}$$ **Ans.**

SECTION B [40 Marks]

*Answer any **four** Questions in this Section.*

Question 5.

(a) *A shopkeeper bought a TV at a discount of 30% of the listed price of ₹ 24,000. The shopkeeper offers a discount of 10% of the listed price to his customer. If the VAT (Value Added Tax) is 10%.*

Find : (i) *the amount paid by the customer.*

(ii) *the VAT to be paid by the shopkeeper.* **[3]**

(b) *Solve the following quadratic equation and give the answer correct to two significant figures.*

$$4x^2 - 7x + 2 = 0$$ **[3]**

(c) *Use graph paper to answer this question.*

(i) *Plot the points A (4, 6) and B (1, 2)*

(ii) *A′ is the image of A when reflected in X-axis.*

(iii) *B′ is the image of B when B is reflected in the line AA′.*

(iv) *Give the geometrical name for the figure AB A′B′.* **[4]**

Solution :

(a) (i) Discount = 30% on 24,000

$$= \frac{30}{100} \times 24{,}000 = ₹\ 7{,}200$$

Cost price of shopkeeper = 24,000 − 7,200

= ₹ 16,800

$$\text{Tax @ } 10\% = \frac{10 \times 16800}{100} = ₹\ 1680$$

Amount paid by shopkeeper = 16,800 + 1,680 = ₹ 18,480

Discount on customer = 10% on 24,000 = $\frac{10}{100} \times 24{,}000$

= ₹ 2,400

Selling price of shopkeeper = 24,000 – 2,400 = ₹ 21,600

Tax @ 10% = $\frac{10 \times 21{,}600}{100}$ = ₹ 2,160

The amount paid by customer = 21,600 + 2,160 = ₹ 23,760 **Ans.**

(ii) VAT to be paid by shopkeeper = ₹ 2,160 – ₹ 1,680

= ₹ 480 **Ans.**

(b) Given equation $4x^2 - 7x + 2 = 0$ comparing with $ax^2 + bx + c = 0$, we have $a = 4$, $b = -7$, $c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+7 \pm \sqrt{(7)^2 - 4 \times 4 \times 2}}{2 \times 4} = \frac{+7 \pm \sqrt{49 - 32}}{8}$$

$$= \frac{7 \pm 4{\cdot}123}{8}$$

Taking $+ve$ sign

$$x = \frac{7 + 4{\cdot}123}{8} = \frac{11{\cdot}123}{8} = 13{\cdot}9$$

Taking $-ve$ sign

$$x = \frac{7 - 4{\cdot}123}{8} = \frac{2{\cdot}877}{8} = 0{\cdot}36$$

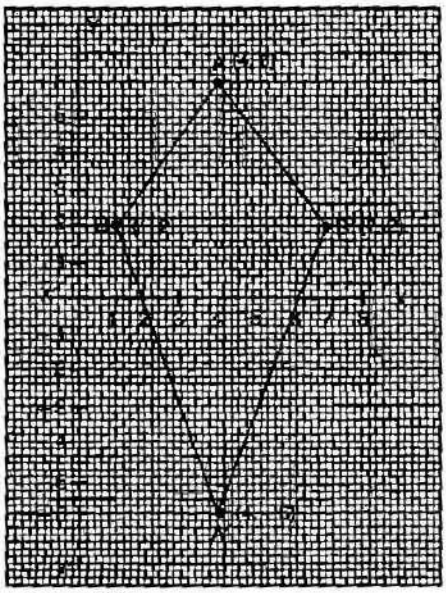
$\Rightarrow$ = 1·390 and 0·3596

= 1·4 and 0·36 **Ans.**

(c) (i) See figure.

(ii) See figure.

(iii) See figure.



(iv) **Geometrical name is Kite.**

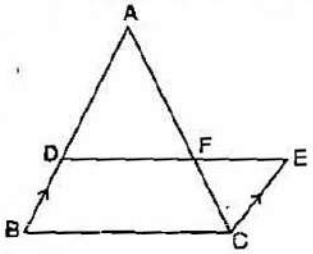
Question 6.

(a) *In the given figure, ABC and CEF are two triangles where BA is parallel to CE and AF : AC = 5 : 8.*

(i) *Prove that Δ ADF ~ Δ CEF.*

(ii) *Find AD if CE = 6 cm.*

(iii) *If DF is parallel to BC find area of Δ ADF : area of Δ ABC.* **[3]**



(b) *Prove the following identity :*

$$\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2\,\text{cosec}\,A$$ **[3]**

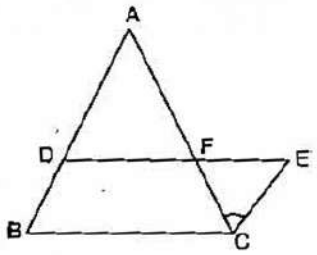
(c) *The following table gives the wages of workers in a factory :*

Wages in ₹	*45–50*	*50–55*	*55–60*	*60–65*	*65–70*	*70–75*	*75–80*
No. of workers	*5*	*8*	*30*	*25*	*14*	*12*	*6*

Calculate the mean by the short cut method. **[4]**

Solution :

(a) In Δ ABC and Δ CEF,



$$BA \parallel CE \text{ (given)}$$

and $$\frac{AF}{AC} = \frac{5}{8} \text{ (given)}$$

$$\Rightarrow \quad \frac{AF}{AF+FC} = \frac{5}{5+3}$$

$$\Rightarrow \quad \frac{AF}{FC} = \frac{5}{3}$$

(i) $\angle DAF = \angle FCE$ (Int. ∠)

$\angle AFD = \angle CFE$ (Vert.)

$\Delta ADF \sim \Delta CEF$ (AA similarity)

Proved

(ii) $$\frac{AD}{CE} = \frac{AF}{FC} \quad \Rightarrow \quad \frac{AD}{6} = \frac{5}{3}$$

$$\Rightarrow \quad AD = \frac{5}{3} \times 6 = 10 \text{ cm}$$ **Ans.**

(iii) *Given* : DF ∥ BC

$$\Delta ADF \sim \Delta ABC$$

$$\frac{AF}{AC} = \frac{AD}{AB} = \frac{5}{8}$$

$$\frac{\text{Area of } \Delta ADF}{\text{Area of } \Delta ABC} = \frac{AD^2}{AB^2} = \frac{(5)^2}{(8)^2} = \frac{25}{64}$$ **Ans.**

(b)

$$\text{L.H.S.} = \frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A}$$

$$= \frac{\sin^2 A + 1 + \cos^2 A + 2\cos A}{\sin A\,(1+\cos A)}$$

$$= \frac{2+2\cos A}{\sin A\,(1+\cos A)} = \frac{2(1+\cos A)}{\sin A(1+\cos A)}$$

$$= 2\,\text{cosec}\,A = \text{R.H.S.}$$

Proved

(c)

Class interval	*Class Marks* x_i	$d_i = x_i - A$	*Frequency* f_i	$f_i.d_i$
45–50	47·5	– 15	5	– 75
50–55	52·5	–10	8	– 80
55–60	57·5	–5	30	– 150
60–65	62·5 (A)	0	25	0
65–70	67·5	5	14	70
70–75	72·5	10	12	120
75–80	77·5	15	6	90
			$\Sigma f_i = 100$	$\Sigma f_i d_i = -25$

Here, A = 62·5

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 62{\cdot}5 + \left(\frac{-25}{100}\right)$$

$$= 62{\cdot}5 - 0{\cdot}25 = 62{\cdot}25$$

Ans.

Question 7.

(a) *Amit Kumar invests ₹ 36,000 in buying ₹ 100 shares at ₹ 20 premium. The dividend is 15% per annum. Find :*

(i) *The number of shares he buys*
(ii) *His yearly dividend*
(iii) *The percentage return on his investment.*

Give your answer correct to the nearest whole number. **[3]**

(b) *What sum of money will amount to ₹ 9,261 in 3 years at 5% per annum compound interest ?* **[3]**

(c) *Mr. Mishra has a Savings Bank Account in Allahabad Bank. His pass book entries are as follows :*

Date	*Particulars*	*Withdrawals (in ₹)*	*Deposits (in ₹)*	*Balance (in ₹)*
Jan. 4, 2007	*By Cash*	—	*1000.00*	*1000.00*
Jan. 11, 2007	*By Cheque*	—	*3000.00*	*4000.00*
Feb. 3, 2007	*By Cash*	—	*2500.00*	*6500.00*
Feb. 7, 2007	*To Cheque*	*2000.00*	—	*4500.00*
March 3, 2007	*By Cash*	—	*5000.00*	*9500.00*
March 25, 2007	*By Cash*	—	*2000.00*	*11,500.00*
June 7, 2007	*By Cash*	—	*3500.00*	*15000.00*
Aug. 29, 2007	*To Cheque*	*1000.00*	—	*14000.00*

Rate of interest paid by the bank is 4.5% per annum. Mr. Mishra closes his account on 30th October, 2007. Find the interest he receives. **[4]**

Solution :

(a) MV of 1 share = ₹ (100 + 20) = ₹ 120

Given : Dividend = 15%, Investment = 36,000

(i) Number of shares buys = $\frac{\text{Investment}}{\text{MV}}$

$= \frac{36,000}{120} = 300$ **Ans.**

(ii) Dividend on 1 share = ₹ $\frac{15}{100} \times 100$ = ₹ 15

Dividend on 300 shares = 15 × 300

= ₹ 4,500 **Ans.**

(iii) Rate of interest = $\frac{4,500}{36,000} \times 100$

= 12·5% **Ans.**

(b) *Given*, A = ₹ 9,261, T = 3 year, R = 5%

$$A = P\left(1 + \frac{R}{100}\right)^T$$

$$\Rightarrow \quad 9,261 = P\left(1 + \frac{5}{100}\right)^3$$

$$\Rightarrow \quad 9,261 = P\left(\frac{21}{20}\right)^3$$

$$\Rightarrow \quad P = \frac{9,261 \times 20 \times 20 \times 20}{21 \times 21 \times 21}$$

= ₹ 8,000 **Ans.**

(c) Qualifying amounts for interest for various months

January	₹	1,000
February	₹	4,500
March	₹	9,500
April	₹	9,500
May	₹	9,500
June	₹	15,000
July	₹	15,000
August	₹	14,000
September	₹	14,000
	₹	92,000

P = ₹ 92,000

T = 1 month

R = 4·5%

Interest = $\frac{P \times R \times T}{100}$

$= \frac{92,000 \times 4·5 \times 1}{100 \times 12}$

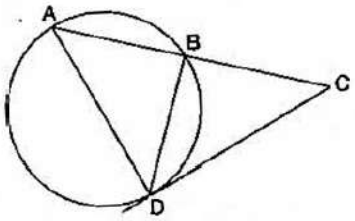
= ₹ 345 **Ans.**

Question 8.

(a) *Given that* $\frac{a^3 + 3ab^2}{b^3 + 3a^2b} = \frac{63}{62}$.

Using Componendo and Dividendo find $a : b$. **[3]**

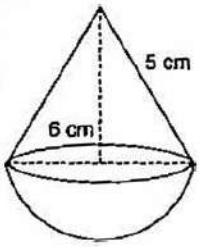
(b)



In the above figure $AB = 7$ *cm and* $BC = 9$ *cm.*

(i) *Prove* $\Delta ACD \sim \Delta DCB$. (ii) *Find the length of* CD. **[3]**

(c) *The given figure represents a hemisphere surmounted by a conical block of wood. The diameter of their bases is 6 cm each and the slant height of the cone is 5 cm. Calculate :*



(i) *the height of the cone.*

(ii) *the volume of the solid.* **[4]**

Solution :

(a) *Given* : $\frac{a^3 + 3ab^2}{b^3 + 3a^2b} = \frac{63}{62}$

By componendo and dividendo,

$$\frac{a^3 + 3ab^2 + b^3 + 3a^2b}{a^3 + 3ab^2 - b^3 - 3a^2b} = \frac{63 + 62}{63 - 62}$$

$$\Rightarrow \quad \frac{(a+b)^3}{(a-b)^3} = \frac{125}{1} = \left(\frac{5}{1}\right)^3$$

$$\Rightarrow \quad \frac{a+b}{a-b} = \frac{5}{1}$$

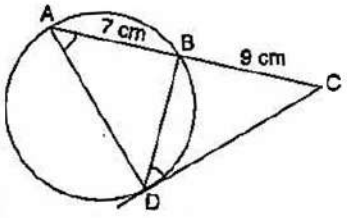
Again componendo and dividendo,

$$\Rightarrow \quad \frac{a+b+a-b}{a+b-a+b} = \frac{5+1}{5-1} = \frac{6}{4}$$

$$\Rightarrow \quad \frac{2a}{2b} = \frac{3}{2}$$

$$\Rightarrow \quad a : b = 3 : 2$$ **Ans.**

(b) *Given* : AB = 7 cm, BC = 9 cm.



(i) In Δ ACD and Δ DCB,

$\angle C = \angle C$ (common)

$\angle CDB = \angle BAD$ ($\angle$'s alternate segment)

$\Delta ACD \sim \Delta DCB$ (By A·A similarity)

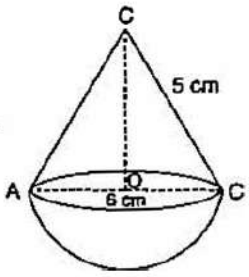
Proved

(ii) $AC \times BC = CD^2$ ($\because \Delta ACD \sim \Delta DCB$)

$CD^2 = 16 \times 9 = 144$

$CD = 12$ cm. **Ans.**

(c) *Given* : Diameter (AB) = 6 cm, r (OB) = 3 cm, l (DB) = 5 cm.



(i) $OD^2 = DB^2 - OB^2$

$= 25 - 9 = 16$

$OD = 4$ cm **Ans.**

(ii) Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (3)^3$

$= 56{\cdot}57$ cm^3.

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 9 \times 4 = 37{\cdot}71$ cm^3

Volume of the solid $= 37{\cdot}71 + 56{\cdot}57$

$= 94{\cdot}28$ cm^3. **Ans.**

Question 9.

(a) *Attempt this question on graph paper.*

Marks obtained by 200 students in examination are given below

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of Students	5	10	14	21	25	34	36	27	16	12

Draw an Ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis.

From the graph find

(i) *the Median*

(ii) *the Upper Quartile*

(iii) *Number of students scoring above 65 marks.*

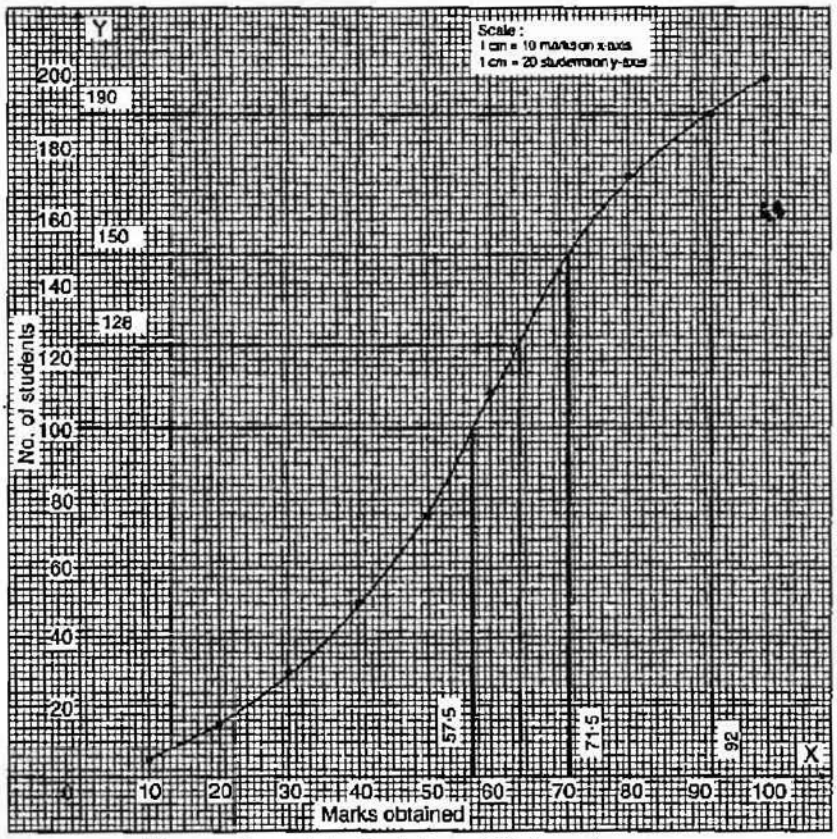
(iv) *If 10 students qualify for merit scholarship, find the minimum marks required to qualify.* **[6]**

(b) *From two points A and B on the same side of a building, the angles of elevation of the top of the building are 30° and 60° respectively. If the height of the building is 10m, find the distance between A and B correct to two decimal places.* **[4]**

Solution :

(a)

Marks	No. of students	*cf*
0–10	5	5
10–20	10	15
20–30	14	29
30–40	21	50
40–50	25	75
50–60	34	109
60–70	36	145
70–80	27	172
80–90	16	188
90–100	12	200



(i) Here n (no. of students) = 200 (even)

$\therefore$ Median = $\left(\frac{n}{2}\right)^{th}$ term = 100th term.

From the graph 100th term is 57·5.

Median = 57·5 **Ans.**

(ii) Upper quartile (Q_3) = $\frac{3n}{4} = \frac{3 \times 200}{4}^{th}$ term

$= 3 \times 50 = 150$ term

From Graph 150^{th} term = 71·5

The upper quartile = 71·5 **Ans.**

(iii) Number of students scoring above 65 marks = 200 − 128 = 72 **Ans.**

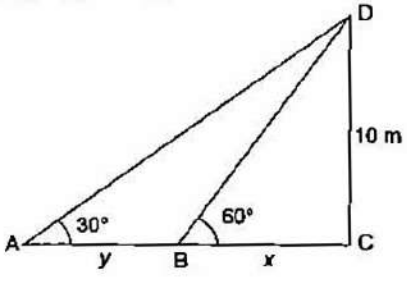
(iv) Minimum marks to qualify = 92 **Ans.**

(b) In Δ DBC, $\tan 60° = \frac{10}{x}$

$\Rightarrow \sqrt{3} = \frac{10}{x}$

$\Rightarrow x = \frac{10}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{10\sqrt{3}}{3}$ m



In Δ DAC, $\tan 30° = \frac{10}{x+y} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad x + y = 10\sqrt{3}$

$\Rightarrow \quad y = 10\sqrt{3} - \frac{10}{\sqrt{3}}$

$= \frac{30-10}{\sqrt{3}} = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{20}{3}\sqrt{3}$

$= 11{\cdot}55$ m **Ans.**

Question 10.

(a) *Mrs. Goswami deposits ₹ 1000 every month in a recurring deposit account for 3 years at 8% interest per annum. Find the matured value.* **[3]**

(b) *Find the equation of a line with x intercept = 5 and passing through the point (4, –7).* **[3]**

(c) *In a school the weekly pocket money of 50 students is as follows :*

Weekly pocket money in ₹	*40-50*	*50-60*	*60-70*	*70-80*	*80-90*	*90-100*
No. of students	*2*	*8*	*12*	*14*	*8*	*6*

Draw a histogram and a frequency polygon on the same graph. Find the mode from the graph. **[4]**

Solution :

(a) Total Principal (P) for 1 month

$P = x \times \frac{n(n+1)}{2} = 1000 \times \frac{36 \times 37}{2}$

$= ₹\,6,66,000$

Interest for 1 month $= \frac{PRT}{100} = \frac{6,66,000 \times 8 \times 1}{100 \times 12}$

$= ₹\,4,440$

Total amount deposited by Mr. Goswami $= 36 \times 1000 = ₹\,36,000$

Maturity value $= ₹\,36,000 + 4,440$

$= ₹\,40,440$ **Ans.**

(b) Equation of the line passing through (5, 0) and (4, –7) :

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{-7-0}{4-5}(x-5)$

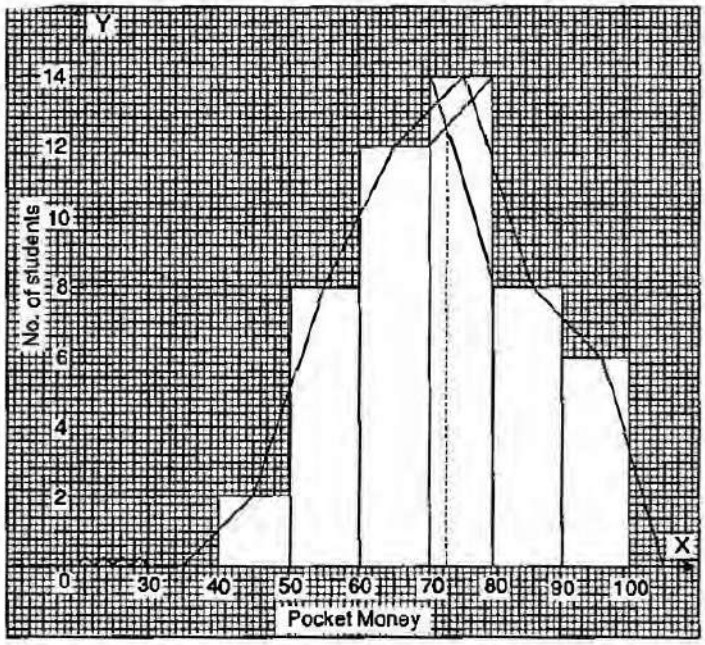
$y = \frac{-7}{-1}(x-5)$

$\Rightarrow \quad y = 7(x-5)$

$\Rightarrow \quad y = 7x - 35$

$\Rightarrow \quad 7x - y - 35 = 0$ **Ans.**

(c)



So, from the graph, Mode = 73 **Ans.**

Question 11.

(a) *The model of a building is constructed with scale factor 1 : 30.*

(i) *If the height of the model is 80 cm, find the actual height of the building in metres.*

(ii) *If the actual volume of a tank at the top of the building is 27m³, find the volume of the tank on the top of the model.* **[2]**

(b) *The speed of an express train is x km/h and the speed of an ordinary train is 12 km/h less than that of the express train. If the ordinary train takes one hour longer than the express train to cover a distance of 240 km, find the speed of the express train.* **[4]**

(c) *Using ruler and compasses construct*

(i) *a triangle ABC in which AB = 5.5 cm, BC = 3.4 cm and CA = 4.9 cm.*

(ii) *the locus of points equidistant from A and C.*

(iii) *a circle touching AB at A and passing through C.* **[4]**

Solution :

(a) Scale factor $k = \frac{1}{30}$

(i) Height of the model = k (times the height of the building)

$\Rightarrow$ Height of building = 80×30

= 2400 cm

= 24 m **Ans.**

(ii) Volume of model = k^3 times volume of the building

$$= \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} \times 27 \text{ m}^3$$

$$= \frac{1}{1000} \text{ m}^3 = 0{\cdot}001 \text{ m}^3$$ **Ans.**

(b) Time taken by express train = $\frac{240}{x}$ h

Speed of ordinary train = $(x - 12)$ km/h

Time taken by ordinary train = $\frac{240}{x - 12}$

According to the question, $\frac{240}{x} = \frac{240}{x - 12} - 1$

$\Rightarrow 240(x - 12) = 240x - x(x - 12)$

$\Rightarrow 240x - 2880 = 240x - x^2 + 12x$

$\Rightarrow x^2 - 12x - 2880 = 0$

$\Rightarrow x^2 - 60x + 48x - 2880 = 0$

$\Rightarrow x(x - 60) + 48(x - 60) = 0$

$\Rightarrow (x - 60)(x + 48) = 0$

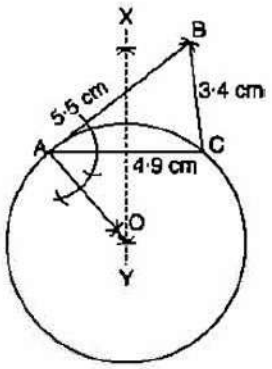
either $x - 60 = 0$ or $x + 48 = 0$

$\Rightarrow x = 60$ or $x = -48$ **Ans.**

Hence, the speed of express train is 60 km/h.

(c) Steps of construction :

1. Draw ΔABC with given values.
2. Draw XY perpendicular bisector of AC.
3. Draw perpendicular of AB at A which cuts perpendicular XY at O.
4. Draw a circle at centre O which touching AB at A and passing through C *i.e.*, required circle.



●●

ICSE Question Paper (2010)

MATHEMATICS

SECTION A [40 Marks]

*(Answer **all** questions from this Section.)*

Question 1.

(a) *Solve the following inequation and represent the solution set on the number line.*

$$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6},\ x \in R \qquad \textbf{[3]}$$

(b) *Tarun bought an article for ₹ 8,000 and spent ₹ 1,000 for transportation. He marked the article at ₹ 11,700 and sold it to a customer. If the customer had to pay 10% sales tax, find*

(i) *The customer's price.*

(ii) *Tarun's profit percent.* **[3]**

(c) *Mr. Gupta opened a recurring deposit account in a bank. He deposited ₹ 2,500 per month for two years. At the time of maturity he got ₹ 67,500. Find :*

(i) *the total interest earned by Mr. Gupta.*

(ii) *the rate of interest per annum.* **[4]**

Solution :

(a) *Given* : $-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6},\ x \in R$

$$-3 < -\frac{1}{2} - \frac{2x}{3} \quad \text{and} \quad -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}$$

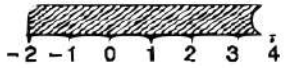
$$-3 + \frac{1}{2} < -\frac{2x}{3} \quad \text{and} \quad -\frac{2x}{3} \leq \frac{5}{6} + \frac{1}{2}$$

$$-\frac{5}{2} < -\frac{2x}{3} \quad \text{and} \quad -\frac{2x}{3} \leq \frac{4}{3}$$

$$\frac{5}{2} > \frac{2x}{3} \quad \text{and} \quad -x \leq 2$$

$$x < \frac{15}{4} \quad \text{and} \quad x \geq -2$$

$$\text{Solution set} = \left\{x : \frac{15}{4} > x \geq -2\right\}$$



(b) *Given* : C.P. = ₹ 8,000 + ₹ 1,000 = ₹ 9,000, M.P. = ₹ 11,700, S.T. = 10%.

(i) Amount to be paid = M.P. + S.T. % of M.P.

$$= 11{,}700 + \frac{10}{100} \times 11{,}700$$

$$= ₹\ 12{,}870 \qquad \textbf{Ans.}$$

(ii) $\text{Profit} = \text{M.P.} - \text{C.P.} = 11{,}700 - 9{,}000$

$= ₹\,2{,}700.$

$\text{Profit percent} = \dfrac{\text{Profit}}{\text{C.P.}} \times 100$

$= \dfrac{2{,}700}{9{,}000} \times 100$

$= 30\%.$

(c) $\text{Total amount deposited} = ₹\,(2{,}500 \times 24) = ₹\,60{,}000$

$\text{Equivalent principal for one month} = ₹\,2{,}500 \times \dfrac{24\left(\frac{24}{2} + 1\right)}{} = ₹\,(62{,}500 \times 12)$

(i) $\text{Total interest} = 67{,}500 - 60{,}000$

$= ₹\,7{,}500$

(ii) Interest on ₹ $(62{,}500 \times 12)$ for 1 month

$= ₹\left(62{,}500 \times 12 \times \dfrac{R}{100} \times \dfrac{1}{12}\right) \quad \left[\because I = \dfrac{P \times R \times T}{100}\right]$

$7{,}500 = 625\,R.$

$\Rightarrow \quad R = 12\%.$ **Ans.**

Question 2.

(a) *Given* $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ *and* $D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$.

Find $AB + 2C - 4D$. **[3]**

(b) *Nikita invests ₹ 6,000 for two years at a certain rate of interest compounded annually. At the end of the first year it amounts to ₹ 6,720. Calculate :*

(i) *the rate of interest.*

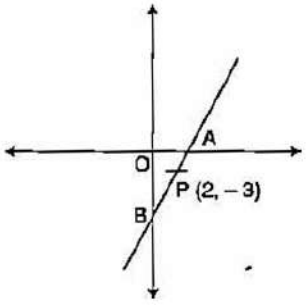
(ii) *the amount at the end of the second year.* **[3]**

(c) *A and B are two points on the x-axis and y-axis respectively. P (2, –3) is the mid point of AB. Find the*

(i) *Coordinates of A and B.*

(ii) *Slope of line AB.*

(iii) *Equation of line AB.* **[4]**



Solution :

(a) *Given* : $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$, $D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$,

$AB + 2C - 4D = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 6 \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} - 4\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 16 \\ -2 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$ **Ans.**

(b) (i) *Given* : Principal = ₹ 6,000, Time = 2 year, After one year amount = ₹ 6,720.

For Ist year : $P + I = ₹ 6,720$

$$6,000 + \frac{P \times R \times 1}{100} = 6,720$$

$$\frac{6,000 \times R}{100} = 720$$

$$\Rightarrow \quad R = 12\% \qquad \textbf{Ans.}$$

(ii) $$A = P\left(1 + \frac{r}{100}\right)^n$$

$$\text{Amount at the end of 2nd year} = 6,000\left(1 + \frac{12}{100}\right)^2$$

$$= 6,000\left(1 + \frac{3}{25}\right)^2$$

$$= 6,000\left(\frac{28}{25} \times \frac{28}{25}\right) = \frac{37,632}{5}$$

$$= ₹ 7,526{\cdot}40. \qquad \textbf{Ans.}$$

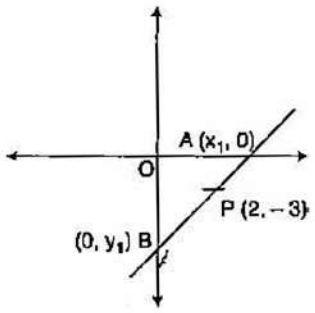
(c) *Given* : A $(x_1, 0)$, B $(0, y_1)$

(i) Mid point coordinates

$$\frac{x_1 + 0}{2} = 2 \qquad x_1 = 4$$

$$\frac{0 + y_1}{2} = -3 \quad \Rightarrow \quad y_1 = -6$$

Coordinates of A (4, 0) and B (0, –6) **Ans.**



(ii) Slope of line AB

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-6 - 0}{0 - 4} = \frac{-6}{-4} = \frac{3}{2} \qquad \textbf{Ans.}$$

(iii) Equation of line $y - y_1 = m(x - x_1)$

$$y - 0 = \frac{3}{2}(x - 4)$$

$$2y = 3x - 12$$

$$3x - 2y - 12 = 0 \qquad \textbf{Ans.}$$

Question 3.

(a) *Cards marked with numbers 1, 2, 3, 4 ... 20 are well shuffled and a card is drawn at random. What is the probability that the number of the cards is*

(i) *a prime number*

(ii) *divisible by 3*

(iii) *a perfect square ?* **[3]**

(b) *Without using trigonometric tables evaluate :*

$$\frac{\sin 35° \cos 55° + \cos 35° \sin 55°}{\text{cosec}^2 10° - \tan^2 80°} \qquad \textbf{[3]}$$

(c) *(Use graph paper for this question)*
A(0, 3), B(3, –2) and O(0, 0) are the vertices of triangle ABO.
(i) *Plot the triangle on a graph sheet taking 2 cm = 1 unit on both the axes.*
(ii) *Plot D the reflection of B in the Y axis, and write its co-ordinates.*
(iii) *Give the geometrical name of the figure ABOD.*
(iv) *Write the equation of the line of symmetry of the figure ABOD.* **[4]**

Solution :

(a) *Given* : Cards marked with numbers 1, 2, ... 20.

$n(S) = 20$

(i) Prime Numbers = 2, 3, 5, 7, 11, 13, 17, 19

$n(E) = 8$

$$P(\text{Prime number}) = P(A) = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$ **Ans.**

(ii) No. divided by 3 = 3, 6, 9, 12, 15, 18

$n(E) = 6$

$$P(\text{no. divided by 3}) = P(A) = \frac{n(E)}{n(S)} = \frac{6}{20} = \frac{3}{10}$$ **Ans.**

(iii) No. perfect square = 1, 4, 9, 16

$n(E) = 4$

$$P(\text{Perfect square}) = P(A) = \frac{n(E)}{n(S)} = \frac{4}{20} = \frac{1}{5}$$ **Ans.**

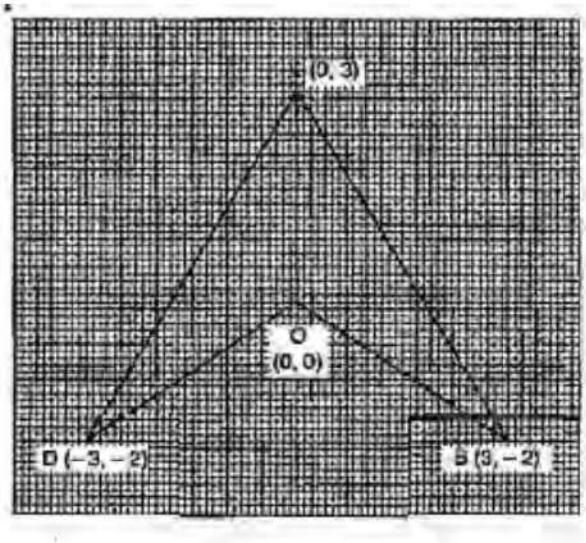
(b) *Given* : $\dfrac{\sin 35° \cos 55° + \cos 35° \sin 55°}{\operatorname{cosec}^2 10° - \tan^2 80°}$

$$= \frac{\sin (90 - 55)° \cos 55° + \cos (90 - 55)° \sin 55°}{\operatorname{cosec}^2 10° - \tan^2 (90 - 10)°}$$

$$\frac{\cos 55° \cos 55° + \sin 55° \sin 55°}{(1 + \cot^2 10°) - \cot^2 10°}$$

$$= \frac{\cos^2 55° + \sin^2 55°}{1 + \cot^2 10° - \cot^2 10°} = \frac{1}{1} = 1$$ **Ans.**

(c) (i) See graph.



(ii) Coordinate of D = (–3, –2)

(iii) Geometrical name of ABOD is arrow.

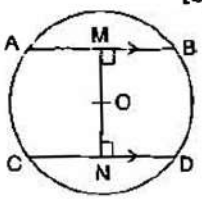
(iv) Equation of the line of symmetry is

$$x = 0$$

Question 4.

(a) *When divided by $x - 3$ the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of 'p'.* **[3]**

(b) *In the figure given alongside AB and CD are two parallel chords and O is the centre. If the radius of the circle is 15 cm, find the distance MN between the two chords of length 24 cm and 18 cm respectively.* **[3]**



(c) *The distribution given below shows the marks obtained by 25 students in an aptitude test. Find the mean, median and mode of the distribution.* **[4]**

Marks obtained	5	6	7	8	9	10
No. of students	3	9	6	4	2	1

Solution :

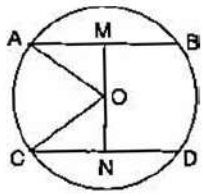
(a) *Given* :

$$f(x) = x^3 - px^2 + x + 6$$
$$g(x) = 2x^3 - x^2 - (p + 3)x - 6$$

when $f(x)$ is divided by $(x - 3)$ remainder $f(3)$ and $f(x)$ is divided by $(x - 3)$ remainder $g(3)$.

$$f(3) = g(3)$$
$$(3)^3 - (3)^2 p + 3 + 6 = 2(3)^3 - (3)^2 - (p + 3)3 - 6$$
$$27 - 9p + 3 + 6 = 54 - 9 - (p + 3)3 - 6$$
$$36 - 9p = 30 - 3p$$
$$9p - 3p = 36 - 30$$
$$6p = 6$$
$$\Rightarrow \quad p = 1 \qquad \textbf{Ans.}$$

(b) *Given* : OA = OC = 15 cm, AB = 24 cm, CD = 18 cm.



Now AM = 12, CN = 9

In Δ OAM, $OA^2 = OM^2 + AM^2$

$$OM^2 = OA^2 - AM^2$$
$$= 15^2 - 12^2$$
$$= 225 - 144$$
$$= 81$$
$$OM = 9$$

Similarly, in Δ OCN, $OC^2 = ON^2 + CN^2$

$$ON^2 = OC^2 - CN^2 = 15^2 - 9^2$$
$$= 225 - 81 = 144$$
$$ON = 12$$
$$MN = OM + ON = 9 + 12 = 21 \text{ cm.} \qquad \textbf{Ans.}$$

(c)

x_i	f_i	x_if_i	cf
5	3	15	3
6	9	54	12
7	6	42	18
8	4	32	22
9	2	18	24
10	1	10	25
	$\Sigma f = 25$	$\Sigma x_if_i = 171$	

$$\text{Mean} = \frac{\Sigma x_i f_i}{N} = \frac{171}{25} = 6{\cdot}84 \quad \textbf{Ans.}$$

$n = 25$ (odd)

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{term} = 13^{\text{th}} \text{ term} = 7 \quad \textbf{Ans.}$$

$$\text{Mode} = 6 \text{ (maximum freq.)} \quad \textbf{Ans.}$$

SECTION B [40 Marks]

*Answer any **four** Questions in this Section.*

Question 5.

(a) *Without solving the following quadratic equation, find the value of 'p' for which the roots are equal.*

$$px^2 - 4x + 3 = 0 \quad \textbf{[3]}$$

(b) *Rohit borrows ₹ 86,000 from Arun for two years at 5% per annum simple interest. He immediately lends out this money to Akshay at 5% compound interest compounded annually for the same period. Calculate Rohit's profit in the transaction at the end of the two years.* **[3]**

(c) *Mrs. Kapoor opened a Saving Bank Account in State Bank of India on 9th January 2008. Her pass book entries for the year 2008 are given below*

Date	Particulars	Withdrawals (in ₹)	Deposits (in ₹)	Balance (in ₹)
Jan 9, 2008	By Cash	—	10,000	10,000
Feb 12, 2008	By Cash	—	15,500	25,500
April 6, 2008	To Cheque	3,500	—	22,000
April 30, 2008	To Self	2,000	—	20,000
July 16, 2008	By Cheque	—	6,500	26,500
Aug. 4, 2008	To Self	5,500	—	21,000
Aug. 20, 2008	To Cheque	1,200	—	19,800
Dec. 12, 2008	By Cash	—	1,700	21,500

Mrs. Kapoor closed the account on 31st December, 2008. If the bank pays interest at 4% per annum, find the interest Mrs. Kapoor receives on closing the account. Give your answer correct to the nearest rupee. **[4]**

Solution :

(a) Roots are equal $\Rightarrow$ $b^2 - 4ac = 0$

$$b^2 = 4ac$$

Given : $a = 4p$, -3, c – .

$$16 = 4.p.3$$

$$p = \frac{16}{12} = \frac{4}{3}$$ **Ans.**

(b) *Given* : P = 86,000, R = 5%, T = 2 years.

$$\text{S.I.} = \frac{P \times R \times T}{100} = \frac{86{,}000 \times 5 \times 2}{100} = ₹\,8{,}600$$

$$\text{C.I.} = P\left[\left(1 + \frac{R}{100}\right)^T - 1\right]$$

$$= 86{,}000\left[\left(1 + \frac{5}{100}\right)^2 - 1\right] = 86{,}000\left[\left(\frac{21}{20}\right)^2 - 1\right] = 86{,}000 \times \frac{41}{400} = ₹\,8{,}815$$

Profit = C.I. – S.I. = 8,815 – 8,600

= ₹ 215 **Ans.**

(c) Minimum balance for the month

January	10,000
February	10,000
March	25,500
April	20,000
May	20,000
June	20,000
July	20,000
August	19,800
September	19,800
October	19,800
November	19,800

Principal = ₹ 2,04,700, R = 4%

$$\text{S.I.} = \frac{P \times R \times T}{100} = \frac{2{,}04{,}700 \times 4 \times 1}{100 \times 12}$$

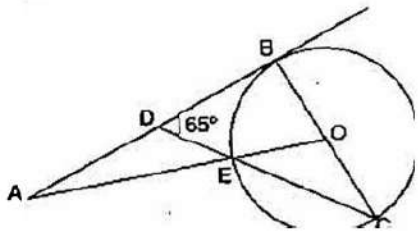
= ₹ 682·33 = ₹ 682. **Ans.**

Question 6.

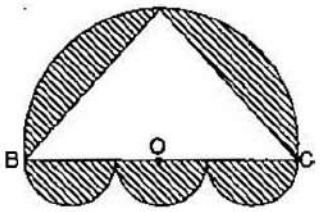
(a) *A manufacturer marks an article for ₹ 5,000. He sells it to a wholesaler at a discount of 25% on the market price and the wholesaler sells it to a retailer at a discount of 15% on the market price. The retailers sells it to a consumer at the market price and at each stage the VAT is 8%. Calculate the amount of VAT received by the Government from :*

(i) *the wholesaler* (ii) *the retailer.* **[3]**

(b) *In the following figure O is the centre of the circle and AB is a tangent to it at point B. ∠BDC = 65°. Find ∠BAO.* **[3]**



(c) *A doorway is decorated as shown in the figure. There are four semi-circles. BC, the diameter of the larger semi circle is of length 84 cm. Centres of the three equal semi-circles lie on BC. ABC is an isosceles triangle with AB = AC. If BO = OC, find the area of the shaded region.* $\left(\text{Take } \pi = \frac{22}{7}\right)$ **[4]**



Solution :

(a) *Given :* Cost of manufacturer = ₹ 5,000

S.P. of manufacturer = C.P. of wholesaler

$= 5{,}000 - \frac{25}{100} \times 5{,}000$

$= 5{,}000 - 1{,}250$

= ₹ 3,750

S.P. of wholesaler = C.P. of retailer

$= 5{,}000 - \frac{15}{100} \times 5{,}000$

$= 5{,}000 - 750$

= ₹ 4,250

S.P. of retailer = C.P. of consumer

= ₹ 5,000

(i) VAT by the wholesaler $= \frac{8}{100} \times 3{,}750$

= ₹ 300 **Ans.**

(ii) VAT by retailer $= \frac{8}{100} \times (4{,}250 - 3{,}750)$

$= \frac{8}{100} \times 500$

= ₹ 40. **Ans.**

(b) AB is tangent $\Rightarrow \angle ABO = 90°$

$\angle BDC = 65°$ (given)

$\Rightarrow \angle BCD = 90° - 65° = 25°$

$\angle BOE = 2 \times 25°$ (angle at centre)

$= 50°$

$\angle BAO = 90° - \angle BOE$

$\angle BAO = 90° - 50°$

$= 40°$ **Ans.**

(c) Let AB = AC = x cm.

As angle in semi circle is 90°

i.e., $\angle A = 90°$

In right angled Δ ABC, by Pythagoras theorem, we get

$AB^2 + AC^2 = BC^2$

$x^2 + x^2 = 84^2$

$$2x^2 = 84 \times 84$$
$$x^2 = 84 \times 42$$

Now $\text{Area of } \Delta ABC = \frac{1}{2} \times AB \times AC$

$$= \frac{1}{2} \times 84 \times 42$$
$$= 1764 \text{ cm}^2.$$

$\text{Diameter of semicircle } (2r) = 84 \text{ cm}$

$\text{Radius } (r) = \frac{1}{2} \times 84 = 42 \text{ cm}$

$\text{Area of semicircle} = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 42 \times 42$

$$= 2772 \text{ cm}^2.$$

$\text{Diameter of each (three equal) semicircles} = \frac{1}{3} \times 84 = 28 \text{ cm}.$

$\text{Radius of the 3 equal semicircles} = \frac{1}{2} \times 28 = 14 \text{ cm}.$

$\text{Area of three equal semi circles} = 3 \times \frac{1}{2}\pi r^2$

$$= 3 \times \frac{1}{2} \times \frac{22}{7} \times 14 \times 14$$
$$= 924 \text{ cm}^2.$$

Area of shaded region = Area of semicircles + Area of three equal circles − Area of Δ ABC

$$= 2772 + 924 - 1764$$
$$= 3696 - 1764$$
$$= 1932 \text{ cm}^2.$$

Ans.

Question 7.

(a) *Use ruler and compasses only for this question :*

(i) *Construct ΔABC, where $AB = 3.5$ cm, $BC = 6$ cm and $\angle ABC = 60°$.*

(ii) *Construct the locus of points inside the triangle which are equidistant from BA and BC.*

(iii) *Construct the locus of points inside the triangle which are equidistant from B and C.*

(iv) *Mark the point P which is equidistant from AB, BC and also equidistant from B and C. Measure and record the length of PB.* **[3]**

(b) *The equation of a line is $3x + 4y - 7 = 0$. Find*

(i) *the slope of the line.*

(ii) *the equation of a line perpendicular to the given line and passing through the intersection of the lines $x - y + 2 = 0$ and $3x + y - 10 = 0$.* **[3]**

(c) *The mean of the following distribution is 52 and the frequency of class interval 30-40 is 'f'. Find 'f'.*

Class Interval	*10-20*	*20-30*	*30-40*	*40-50*	*50-60*	*60-70*	*70-80*
Frequency	*5*	*3*	*f*	*7*	*2*	*6*	*13*

[4]

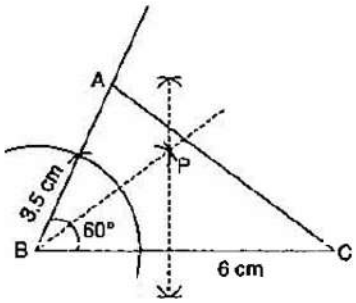
Solution :

(a) Steps of Construction :

(i) Draw BC = 6 cm and make an angle at B = 60°. Cut BA = 3·5 cm and meet A to C. This is the required Δ ABC.

(ii) Draw the bisector of Δ ABC and perpendicular bisector of BC; both intersecting at P.

(iii) P is the required point. PB = 3·5 cm.



(b) *Given* : Equation of the line is

$$3x + 4y - 7 = 0$$

$$4y = -3x + 7$$

$$y = -\frac{3}{4}x + \frac{7}{4}$$

(i) Slope of the line $(m_1) = -\frac{3}{4}$ **Ans.**

(ii) Slope of the perpendicular line $(m_2) = \frac{-1}{m_1} = \frac{-1}{-3/4} = \frac{4}{3}$

Intersection of the lines $x - y + 2 = 0$ ...(i)

and $3x + y - 10 = 0$ ...(ii)

By Adding equation (i) and (ii) $4x = 8$ $x = 2$

Put $x = 2$, in equation (i) we get

$$2 - y + 2 = 0 \Rightarrow y = 4$$

Equation of line $y - y_1 = m_2(x - x_1)$

$$y - 4 = \frac{4}{3}(x - 2)$$

$$4x - 3y + 4 = 0$$ **Ans.**

(c)

Interval	*Frequency* (f_i)	x_i	$d_i = x_i - A$	$f_i d_i$
10–20	5	15	– 30	– 150
20–30	3	25	– 20	– 60
30–40	f	35	– 10	– 10f
40–50	7	45 A	0	0
50–60	2	55	10	20
60–70	6	65	20	120
70–80	13	75	30	390
	$36 + f$			$\Sigma f_i d_i = 320 - 10f$

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{N}$$

$$52 = 45 + \frac{320 - 10f}{36 + f}$$

$$\Rightarrow \quad 7 = \frac{320 - 10f}{36 + f}$$

$$\Rightarrow \quad 252 + 7f = 320 - 10f$$

$$\Rightarrow \quad 17f = 68$$

$$\Rightarrow \quad f = 4 \qquad \textbf{Ans.}$$

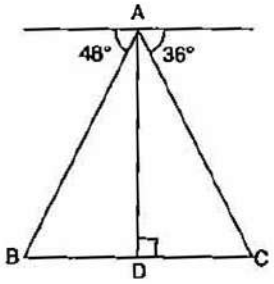
Question 8.

(a) *Use the Remainder Theorem to factorise the following expression :*

$$2x^3 + x^2 - 13x + 6 \qquad [3]$$

(b) *If x, y, z are in continued proportion, prove that* $\frac{(x+y)^2}{(y+z)^2} = \frac{x}{z}$. **[3]**

(c) *From the top of a light house 100 m high the angles of depression of two ships on opposite sides of it are 48° and 36° respectively. Find the distance between the two ships to the nearest metre.* **[4]**



Solution :

(a) *Given* :

$$f(x) = 2x^3 + x^2 - 13x + 6$$
$$f(1) = 2 + 1 - 13 + 6 \neq 0$$
$$f(-1) = -2 + 1 + 13 + 6 \neq 0$$
$$f(2) = 16 + 4 - 26 + 6 = 0$$

So, $x - 2$ is one factor of $f(x)$ by remainder theorem.

$$\begin{array}{r} 2x^2 + 5x - 3 \\ x - 2 \overline{)\ 2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 - 4x^2} \\ \quad 5x^2 - 13x + 6 \\ \quad \underline{5x^2 - 10x} \\ \quad\quad -3x + 6 \\ \quad\quad -3x + 6 \\ \quad\quad\quad \times \end{array}$$

$\therefore$ The other factors of $f(x)$ are the factors of $2x^2 + 5x - 3$.

$$= 2x^2 + 6x - x - 3$$
$$= 2x(x + 3) - 1(x + 3)$$
$$= (2x - 1)(x + 3)$$

Hence, $2x^3 + x^2 - 13x + 6 = (2x - 1)(x + 3)(x - 2)$ **Ans.**

(b) If x, y, z are in continued proportion

$$\frac{x}{y} = \frac{y}{z} = k$$

$\Rightarrow$ $y = kz$

and $x = xy = k^2z$

$$\text{L.H.S.} = \frac{(x+y)^2}{(y+z)^2} = \frac{(k^2z + kz)^2}{(kz+z)^2}$$

$$= \frac{k^2z^2(k+1)^2}{z^2(k+1)^2}$$

$$= k^2$$

$$\text{R.H.S.} = \frac{x}{z} = \frac{k^2z}{z} = k^2$$

Hence L.H.S. = R.H.S. **Proved**

(c) In Δ ABD, $\tan 48° = \frac{AD}{BD}$

$\Rightarrow$ $1{\cdot}11 = \frac{100}{BD}$

$$BD = \frac{100}{1{\cdot}11} = 90{\cdot}09 \text{ m}$$

In Δ ACD, $\tan 36° = \frac{AD}{DC}$

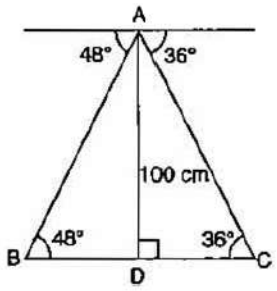
$$0{\cdot}7265 = \frac{100}{DC}$$

$\Rightarrow$ $DC = \frac{100}{0{\cdot}7265} = 137{\cdot}64$ m

$BC = BD + DC$

$= 90{\cdot}09 + 137{\cdot}64$

$= 227{\cdot}73$ m. **Ans.**



Question 9.

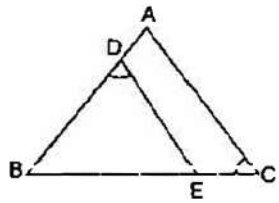
(a) *Evaluate* :

$$\begin{bmatrix} 4\sin 30° & 2\cos 60° \\ \sin 90° & 2\cos 0° \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$ **[3]**

(b) *In the figure ABC is a triangle with* $\angle EDB = \angle ACB$.
Prove that $\Delta ABC \sim \Delta EBD$.
If BE = 6 cm, EC = 4 cm, BD = 5 cm and area of $\Delta BED = 9\ cm^2$. *Calculate the* :

(i) *length of AB*

(ii) *area of* ΔABC. **[4]**



(c) *Vivek invests* ₹ *4,500 in 8%,* ₹ *10 shares at* ₹ *15. He sells the shares when the price rises to* ₹ *30, and invests the proceeds in 12%* ₹ *100 shares at* ₹ *125. Calculate* :

(i) *the sale proceeds.*

(ii) *the number of* ₹ *125 shares he buys.*

(iii) *the change in his annual income from dividend.* **[4]**

Solution :

(a) *Given* :

$$\begin{bmatrix} 4\sin 30° & 2\cos 60° \\ \sin 90° & 2\cos 0° \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4\times\frac{1}{2} & 2\times\frac{1}{2} \\ 1 & 2\times 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$$ **Ans.**

(b) $\angle EDB = \angle ACB$ (given)

$\angle DBE = \angle ABC$

$\angle DEB = \angle BAC$ (AA axiom)

$\Rightarrow \quad \Delta ABC \sim \Delta EBD$ **Proved**

(i) *Given* : BE = 6 cm, EC = 4 cm, BD = 5 cm.

$$\frac{AB}{EB} = \frac{BC}{BD} = \frac{AC}{ED}$$

$$\frac{AB}{EB} = \frac{BC}{BD}$$

$$\frac{AB}{6} = \frac{BE + EC}{5} = \frac{6+4}{5} = \frac{10}{5} = 2$$

$AB = 12$ cm **Ans.**

(ii) $$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta EBD} = \frac{AB^2}{EB^2} = \frac{144}{36}$$

$$\frac{\text{Area of } \Delta ABC}{9} = \frac{(12)^2}{(6)^2}$$

$\Rightarrow \quad$ Area of $\Delta ABC = \frac{144\times 9}{36} = 36$ m. **Ans.**

(c) Number of shares bought $= \frac{4{,}500}{15}$

$= 300$

Total face value $= ₹\,300 \times 10$

$= ₹\,3{,}000$

Dividend $= \frac{8}{100}\times 3{,}000$

$= ₹\,240.$

Amount received on selling 300 shares for ₹ $= 300 \times 30 = ₹\,9{,}000$

(i) Sale proceeds $= ₹\,9{,}000 - ₹\,4{,}500 = ₹\,4{,}500$ **Ans.**

(ii) Number of shares bought at ₹ 125 $= \frac{9{,}000}{125}$

$= 72$ **Ans.**

(iii) Total face value of 72 shares = ₹ 72 × 100

= ₹ 7,200

Dividend = $\frac{12}{100} \times 7{,}200$

= ₹ 864.

Change in his annual income = 864 – 240

= ₹ 624. **Ans.**

Question 10.

(a) *A positive number is divided into two parts such that the sum of the squares of the two parts is 208. The square of the larger part is 8 times the smaller part. Taking x as the smaller part of the two parts, find the number.* **[4]**

(b) *The monthly income of a group of 320 employees in a company is given below :*

Monthly Income	*No. of Employees*
6000–7000	20
7000–8000	45
8000–9000	65
9000–10000	95
10000–11000	60
11000–12000	30
12000–13000	5

Draw an ogive of the given distribution on a graph sheet taking 2 cm = ₹ 1,000 on one axis and 2 cm = 50 employees on the other axis. From the graph determine :

(i) *the median wage.*

(ii) *the number of employees whose income is below ₹ 8,500*

(iii) *If the salary of a senior employee is above ₹ 11,500, find the number of senior employees in the company.*

(iv) *the upper quartile.* **[6]**

Solution :

(a) Let x and y are the two parts.

$$x^2 + y^2 = 208 \quad \ldots(1)$$

$$y^2 = 8x \quad \ldots(2)$$

$$\Rightarrow \quad x^2 + 8x - 208 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 1, b = 8, c = -208$

$$\frac{-8 \pm \sqrt{(8)^2 - 4 \times 1 \times (-208)}}{2 \times 1}$$

$$= \frac{-8 \pm \sqrt{64 + 832}}{}$$

$$= \frac{-8 \pm 29{\cdot}93}{2} = \frac{-8 + 29{\cdot}93}{2} \text{ or } \frac{-8 - 29{\cdot}93}{2}$$

$$= -18{\cdot}96 \text{ or } 10{\cdot}97$$

$$y^2 = 8x$$

$$= 8 \times 10{\cdot}97$$

$$= 87{\cdot}76$$

$$y = 9{\cdot}37$$

$$\text{Number} = x + y$$

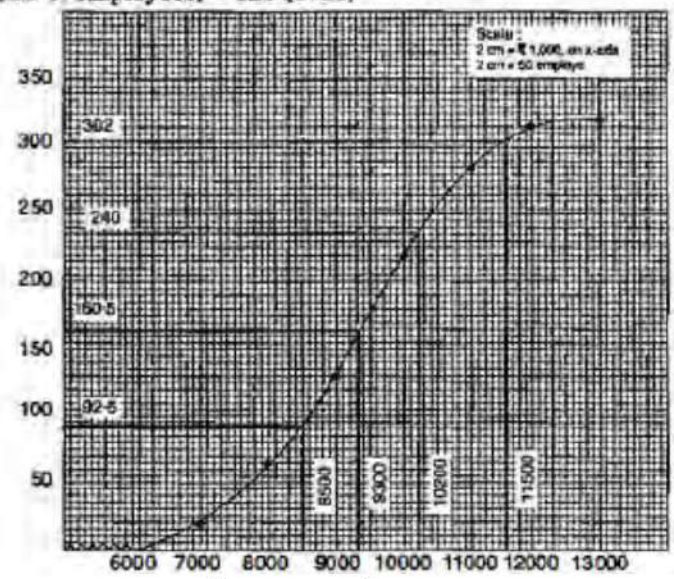
$$= 10{\cdot}97 + 9{\cdot}37$$

$$= 20{\cdot}34$$ **Ans.**

(b)

Monthly Income	*No. of Employees*	*C.F.*
6000–7000	20	20
7000–8000	45	65
8000–9000	65	130
9000–10000	95	225
10000–11000	60	285
11000–12000	30	315
12000–13000	5	320
	320	

Here n (no. of employees) = 320 (even)



(i) $$\text{Median} = \frac{1}{2}\left[\frac{n}{2} + \left(\frac{n}{2} + 1\right)\right] = \frac{1}{2}[160 + 161] = 160{\cdot}5$$

Required median = ₹ 9,300 (from graph) **Ans.**

(ii) Number of employees whose income is below ₹ 8,500 = 92·5 approx. **Ans.**

(iii) Number of senior employees in the company = $320 - 302 = 18$. **Ans.**

(iv) $$\text{Upper quartile} = \frac{3n}{4} = \frac{3 \times 320}{4} = 240$$

$$\text{Upper quartile} = 10{,}200.$$

Question 11.

(a) *Construct a regular hexagon of side 4 cm. Construct a circle circumscribing the hexagon.* **[3]**

(b) *A hemispherical bowl of diameter 7.2 cm is filled completely with chocolate sauce. This sauce is poured into an inverted cone of radius 4.8 cm. Find the height of the cone.* **[3]**

(c) *Given* $x = \dfrac{\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}}{\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2}}$

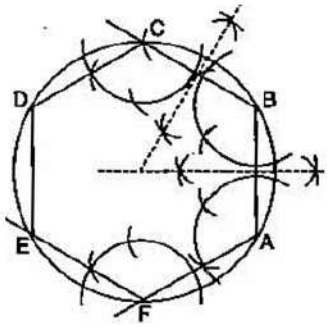
Use componendo and dividendo to prove that $b^2 = \dfrac{2a^2x}{x^2 + 1}$. **[4]**

Solution :

(a) Steps of Construction :

(i) Using the given data, construct the regular hexagon ABCDEF with each side equal to 4 cm.

(ii) Draw the perpendicular bisectors of sides AB and AF which intersect each other at point O.



(iii) With O as centre and OA as radius draw a circle which will pass through all the vertices of the regular hexagon ABCDEF.

(b) *Given* : Diameter of hemispherical bowl = 7·2 cm

Radius of hemispherical bowl = 3·6 cm

$$\text{Volume of hemispherical bowl} = \frac{2}{3} \times \pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 3{\cdot}6 \times 3{\cdot}6 \times 3{\cdot}6$$

$$= 97{\cdot}76 \text{ cm}^3.$$

$$\text{Volume of cone} = \frac{1}{3} \times \pi R^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 4{\cdot}8 \times 4{\cdot}8 \times h$$

$$= 24{\cdot}14 \times h \text{ cm}^3$$

Volume of cone = Volume of hemisperical bowl

$$24{\cdot}14 \times h = 97{\cdot}76$$

$$h = \frac{97{\cdot}76}{24{\cdot}14}$$

$$= 4{\cdot}05 \text{ cm.}$$ **Ans.**

(c) *Given* : $x = \dfrac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}$

Componendo and dividendo

$$\frac{x+1}{x-1} = \frac{(\sqrt{a^2+b^2}+\sqrt{a^2-b^2})+(\sqrt{a^2+b^2}-\sqrt{a^2-b^2})}{(\sqrt{a^2+b^2}+\sqrt{a^2-b^2})-(\sqrt{a^2+b^2}-\sqrt{a^2-b^2})}$$

$$= \frac{2(\sqrt{a^2+b^2})}{2\sqrt{a^2-b^2}}$$

$$\frac{(x+1)^2}{(x-1)^2} = \frac{a^2+b^2}{a^2-b^2}$$

Again componendo and dividendo

$$\frac{(x+1)^2+(x-1)^2}{(x+1)^2-(x-1)^2} = \frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2}$$

$$\frac{2x^2+2}{4x} = \frac{2a^2}{2b^2}$$

$$\frac{x^2+1}{2x} = \frac{a^2}{b^2}$$

$$b^2 = \frac{2a^2x}{x^2+1}$$ **Proved**

ICSE Question Paper (2011)

MATHEMATICS

SECTION A [40 Marks]

*(Answer **all** questions from this Section.)*

Question 1.

(a) *Find the value of 'k' if (x – 2) is a factor of :*

$$x^3 + 2x^2 - kx + 10$$

Hence determine whether (x + 5) is also a factor. **[3]**

(b) *If* $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ *and* $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, *is the product AB possible ? Give a reason. If yes, find AB.* **[3]**

(c) *Mr. Kumar borrowed ₹ 25,000 for two years. The rate of interest for the two successive years are 8% and 10% respectively. If he repays ₹ 6,200 at the end of the first year, find the outstanding amount at the end of the second year.* **[4]**

Solution :

(a) Let $f(x) = x^3 + 2x^2 - kx + 10$

$\because (x - 2)$ is a factor,

$$f(2) = 0$$

$$f(2) = 8 + 8 - 2k + 10 = 0$$

$$k = 13$$

To check for $(x + 5)$ is a factor,

$$f(-5) = (-5)^3 + 2(-5)^2 - 13(-5) + 10$$

$$= -125 + 50 + 65 + 10 = 0$$

$\therefore (x + 5)$ is a factor.

(b) $A_{2\times 2} \cdot B_{2\times 1}$

From the order of both the matrix A and B, it is clear that AB is possible because the number of columns of A are equal to the number of rows of B.

$$AB = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 + 20 \\ 8 - 8 \end{bmatrix}$$

$$= \begin{bmatrix} 26 \\ 0 \end{bmatrix}$$

Ans.

(c) *Given* : Principal = ₹ 15,000

We know that $A = P\left(1 + \frac{r}{100}\right)^n$

Amount after 1st year $= 15{,}000\left(1 + \frac{8}{100}\right)$

$= ₹\ 16{,}200$

Principal after repayment $= 16{,}200 - 6{,}200 = ₹\ 10{,}000$

Amount outstanding at the end of second year

$= 10{,}000\left(1 + \frac{10}{100}\right)$

$= ₹\ 11{,}000$ **Ans.**

Question 2.

(a) *From a pack of 52 playing cards all cards whose numbers are multiples of 3 are removed. A card is now drawn at random.*

What is the probability that the card drawn is

(i) *a face card (King, Jack or Queen)*

(ii) *an even numbered red card ?* **[3]**

(b) *Solve the following equation :*

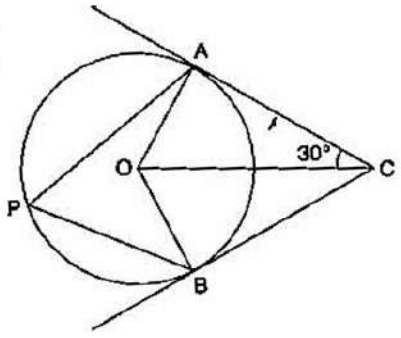
$x - \frac{18}{x} = 6$. *Give your answer correct to two significant figures.* **[3]**

(c) *In the given figure O is the centre of the circle. Tangents at A and B meet at C. If* $\angle AOC = 30°$, *find*

(i) $\angle BCO$

(ii) $\angle AOB$

(iii) $\angle APB$



Solution :

(a) Number of cards which are multiples of 3 = 12

Cards left in the pack = 40

(i) Number of face cards = 12

P (face card) $= \frac{12}{40} = \frac{3}{10}$ **Ans.**

(ii) Even numbered red cards = 10

P (even number red card) $= \frac{10}{40} = \frac{1}{4}$ **Ans.**

(b) Let $x^2 - 6x - 18 = 0$

Compare with equation $ax^2 + bx + c = 0$, we get

$a = 1, b = -6, c = -18$

Now, $x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{6 \pm \sqrt{36 + 72}}{2}$

$$= \frac{6 \pm 6\sqrt{3}}{2} \Rightarrow x = 3 \pm 3\sqrt{3}$$

$$x = 3 \pm 5{\cdot}196$$

Taking *+ve* and *– ve* sign respectively, we get

$x = 8{\cdot}196$ or $x = -2{\cdot}196$ **Ans.**

(c) $\Delta ACO \cong \Delta BCO$ (R.H.S.)

$\angle BCO = \angle ACO$ (C.P.C.T.)

(i) $\angle BCO = 30°$

In ΔACO, $\angle OAC = 90°$ (Radius is perpendicular to tangent)

$\angle AOC = 60°$

Also $\angle BOC = 60°$ (C.P.C.T.)

(ii) $\angle AOB = 120°$

(iii) $\angle APB = 60°$ (Angle at circumference is half the angle at the centre)

Question 3.

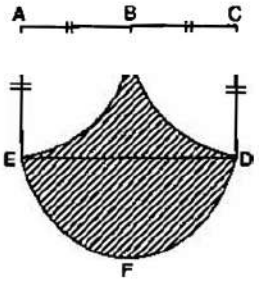
(a) *Ahmed has a recurring deposit account in a bank. He deposits ₹ 2,500 per month for 2 years. If he gets ₹ 66,250 at the time of maturity, find*

(i) *The interest paid by the bank.*

(ii) *The rate of interest.* **[3]**

(b) *Calculate the area of the shaded region, if the diameter of the semi circle is equal to 14 cm.*

Take $\pi = \frac{22}{7}$ **[3]**



(c) *ABC is a triangle and G(4, 3) is the centroid of the triangle. If A = (1, 3), B = (4, b) and C = (a, 1), find 'a' and 'b'. Find the length of side BC.* **[4]**

Solution :

(a) (i) Interest $= 66{,}250 - 2{,}500 \times 24$

$= 66{,}250 - 60{,}000$

$= ₹\,6{,}250$ **Ans.**

(ii) $\text{Principal} = \frac{n(n+1)}{2} \times \text{sum of deposited per month}$

$= \frac{24\,(24+1)}{2} \times 2{,}500$

$= \frac{24 \times 25}{2} \times 2{,}500$

$I = \text{Principal} \times \frac{R}{100} \times \frac{1}{12}$

$$R = \frac{6{,}250 \times 2 \times 100 \times 12}{2{,}500 \times 24 \times 25}$$

$= 10\%$ p.a. **Ans.**

(b) Area of shaded portion = Complete area – area of the two quadrants

= (Area of ACDE + Area of semi circle EFD) – (Area of Quadrant ABE + Area of Quadrant BCD)

$$= \left\{14 \times 7 + \frac{\pi}{2}(7)^2\right\} - \left\{\frac{\pi}{4}(7)^2 + \frac{\pi}{4}(7)^2\right\}$$

$$= \left\{14 \times 7 + \frac{\pi}{2}(7)^2\right\} - \left\{\frac{\pi}{2}(7)^2\right\}$$

$= 98 \text{ cm}^2$. **Ans.**

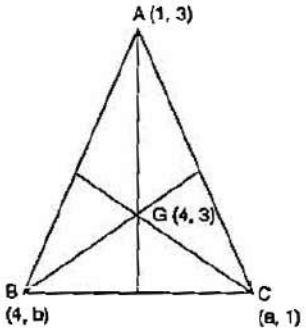
(c) Coordinate of centroid $G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$$\Rightarrow \quad \frac{1 + 4 + a}{3} = 4 \quad \Rightarrow \quad a = 7$$

$$\frac{3 + b + 1}{3} = 3 \quad \Rightarrow \quad b = 5$$

Now, $BC = \sqrt{(4-7)^2 + (5-1)^2}$

$= \sqrt{9 + 16} = 5$ units. **Ans.**



Question 4.

(a) *Solve the following inequation and represent the solution set on the number line* $2x - 5 \le 5x + 4 < 11$, *where* $x \in I$. **[3]**

(b) *Evaluate without using trigonometric tables :*

$$2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right)^2 - 3\left(\frac{\sec 40^\circ}{\text{cosec } 50^\circ}\right)$$ **[3]**

(c) *A Mathematics aptitude test of 50 students was recorded as follows :*

Marks	*50-60*	*60-70*	*70-80*	*80-90*	*90-100*
No. of students	*4*	*8*	*14*	*19*	*5*

Draw a histogram for the above data using a graph paper and locate the mode. **[4]**

Solution :

(a) *Given :* $2x - 5 \le 5x + 4$ and $5x + 4 < 11$

$-3x \le 9$ $\quad 5x < 7$

$x \ge -3$ $\quad x < 1{\cdot}4$

$-3 \le x$

Solution set, $x \in \{-3, -2, -1, 0, 1\}$

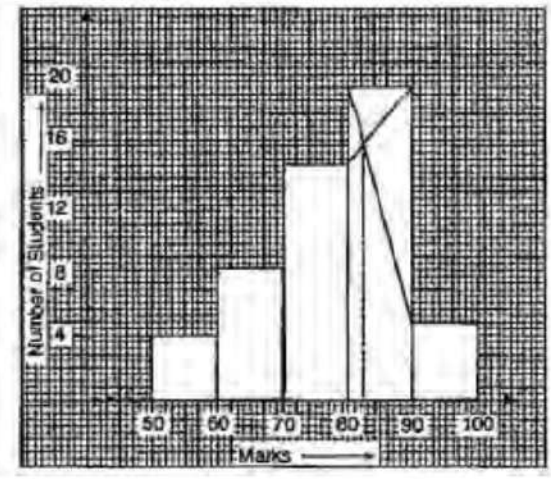
(b) *Given :* $2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right) - 3\left(\frac{\sec 40^\circ}{\text{cosec } 50^\circ}\right)$

$= 2\left(\frac{\tan (90^\circ - 55^\circ)}{\cot 55^\circ}\right)^2 + \left(\frac{\cot (90^\circ - 35^\circ)}{\tan 35^\circ}\right) - 3\left(\frac{\sec (90^\circ - 50^\circ)}{\text{cosec } 50^\circ}\right)$

$= 2\left(\frac{\cot 55^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\tan 35^\circ}{\tan 35^\circ}\right) - 3\left(\frac{\text{cosec } 50^\circ}{\text{cosec } 50^\circ}\right)$

$= 2 + 1 - 3 = 0$ **Ans.**

(c)



Mode from graph = 82·5.

SECTION B [40 Marks]

*Answer any **four** Questions in this Section.*

Question 5.

(a) *A manufacturer sells a washing machine to a wholesaler for ₹ 15,000. The wholesaler sells it to a trader at a profit of ₹ 1,200 and the trader in turn sells it to a consumer at a profit of ₹ 1,800. If the rate of VAT is 8% find :*

(i) *The amount of VAT received by the State Government on the sale of this machine from the manufacturer and the wholesaler.*

(ii) *The amount that the consumer pays for the machine.* **[3]**

(b) *A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0.5 cm. Find the number of spheres formed.* **[3]**

(c) *ABCD is a parallelogram where A(x, y), B (5, 8), C (4, 7) and D (2, –4). Find*

(i) *Coordinates of A*

(ii) *Equation of diagonal BD.* **[4]**

Solution :

(a) (i) VAT received by Govt. from manufacturer $= 15{,}000 \times \frac{8}{100} = ₹\,1{,}200$

VAT from wholesaler $= 1200 \times \frac{8}{100} = ₹\,96$

Total VAT from manufacturer and wholesaler

$= 1200 + 96 = ₹\,1296$ **Ans.**

(ii) Amount that customer pays = (15000 + 1200 + 1800) + VAT

$$= 18{,}000 + \frac{18{,}000 \times 8}{100}$$

$$= 18{,}000 + 1440 = ₹\ 19{,}440$$ **Ans.**

(b) Number of spheres = $\frac{\text{Volume of cone}}{\text{Volume of each sphere}}$

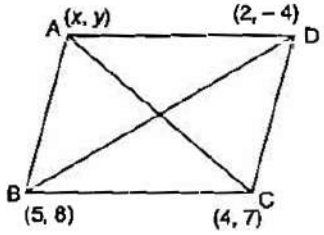
$$= \frac{\frac{1}{3}\pi (5)^2 (8)}{\frac{4}{3}\pi (0{\cdot}5)^3} = \frac{50 \times 10^3}{5 \times 5 \times 5}$$

$$= 400$$ **Ans.**

(c) In a parallelogram, mid point of diagonal BD co-incides with the mid point of diagonal AC.

Mid point of BD $= \left(\frac{5+2}{2}, \frac{8-4}{2}\right) = \left(\frac{7}{2}, 2\right)$

Mid point of AC $= \left(\frac{x+4}{2}, \frac{y+7}{2}\right)$



Equating, $\frac{x+4}{2} = \frac{7}{2} \Rightarrow x = 3$

$\frac{y+7}{2} = 2 \Rightarrow y = -3$

(i) Co-ordinates of A (3, –3). **Ans.**

(ii) m of BD $= \frac{8-(-4)}{5-2} = \frac{12}{3} = 4$

Equation of BD,

$$y - y_1 = m(x - x_1)$$

$$y + 4 = 4(x - 2)$$

$$y = 4x - 4$$ **Ans.**

Question 6.

(a) *Use a graph paper to answer the following questions. (Take 1 cm = 1 unit on both axes).*

(i) *Plot A(4, 4), B (4, –6) and C (8, 0), the vertices of a triangle ABC.*

(ii) *Reflect ABC on the y-axis and name it as A'B'C'.*

(iii) *Write the coordinates of the image A', B' and C'.*

(iv) *Give a geometrical name for the figure AA'C'B'BC.*

(v) *Identify the line of symmetry of AA'C'B'BC.* **[5]**

(b) *Mr. Choudhury opened a Saving's Bank Account at State Bank of India on 1st April 2007. The entries of one year as shown in his pass book are given below :*

Date	*Particulars*	*Withdrawals (in ₹)*	*Deposits (in ₹)*	*Balance (in ₹)*
1st April 2007	*By Cash*	—	*8550.00*	*8550.00*
12th April 2007	*To Self*	*1200.00*	—	*7350.00*
24th April 2007	*By Cash*	—	*4550.00*	*11900.00*

8th July 2007	*By Cheque*		*1500.00*	*13400.00*
10th Sept. 2007	*By Cheque*		*3500.00*	*16900.00*
17th Sept. 2007	*To Cheque*	*2500.00*		*14400.00*
11th Oct. 2007	*By Cash*		*800.00*	*15200.00*
6th Jan. 2008	*To Self*	*2000.00*		*13200.00*
9th March 2008	*By Cheque*		*950.00*	*14150.00*

If the bank pays interest at the rate of 5% per annum, find the interest paid on 1st April, 2008. Give your answer correct to the nearest rupee. **[5]**

Solution :

(a) (i) In the given diagram.

(ii) In the given diagram.

(iii) A′ (–4, 4)

B′ (–4, –6)

C′ (–8, 0)

(iv) Hexagon.

(v) Y–Axis is the line of symmetry.

(b)

	Minimum Balances
April	7,350.00
May	11,900.00
June	11,900.00
July	13,400.00
August	13,400.00
September	14,400.00
October	14,400.00
November	15,200.00
December	15,200.00
January	13,200.00
February	13,200.00
March	14,150.00
	1,57,700.00 ⇒ P = ₹ 1,57,700

$$I = \frac{1,57,700 \times 5}{100} \times \frac{1}{12}$$

$$= \frac{7885}{12} = ₹\ 657.08$$ **Ans.**

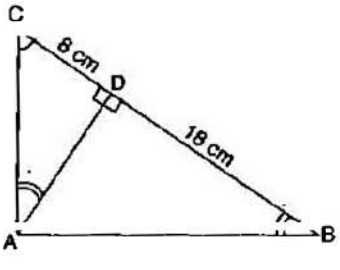
Question 7.

(a) *Using componendo and dividendo, find the value of x*

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = 9$$ **[3]**

(b) *If* $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ *and I is the identity matrix of the same order and* A^1 *is the transpose of matrix A, find* $A^1.B + BI$. **[3]**

(c) *In the adjoining figure ABC is a right angled triangle with* $\angle BAC = 90°$.



(i) *Prove* $\Delta ADB \sim \Delta CDA$.

(ii) *If* $BD = 18$ *cm,* $CD = 8$ *cm, find AD.*

(iii) *Find the ratio of the area of* ΔADB *is to area of* ΔCDA. **[4]**

Solution :

(a) *Given :* $\dfrac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = \dfrac{9}{1}$

Applying componendo and Dividendo,

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}+\sqrt{3x+4}-\sqrt{3x-5}}{\sqrt{3x+4}+\sqrt{3x-5}-\sqrt{3x+4}+\sqrt{3x-5}} = \frac{9+1}{9-1}$$

$$\frac{2\sqrt{3x+4}}{2\sqrt{3x-5}} = \frac{10}{8}$$

$$\frac{\sqrt{3x+4}}{\sqrt{3x-5}} = \frac{5}{4}$$

Squaring both sides,

$$\frac{3x+4}{3x-5} = \frac{25}{16}$$

Applying Componendo and Dividendo,

$$\frac{3x+4+3x-5}{3x+4-3x+5} = \frac{25+16}{25-16}$$

$$\frac{6x-1}{9} = \frac{41}{9}$$

$$6x = 42$$

$$x = 7$$ **Ans.**

(b) Transpose of matrix A, $A^t = \begin{bmatrix} 2 & 1 \\ & 3 \end{bmatrix}$

$$A^t.B = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 8-1 & -4+4 \\ 20-3 & -10+9 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix}$$

$$B.I = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$A^t \cdot B + B.I = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

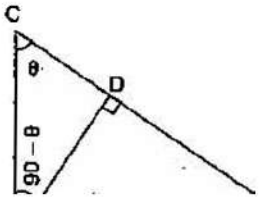
$$= \begin{bmatrix} 11 & -3 \\ 6 & 2 \end{bmatrix}$$ **Ans.**

(c) Let $\angle DAB = \theta$

$\angle DAC = 90 - \theta$

$\angle DBA = 90 - \theta$

$\angle DCA = \theta$



$\because$ All three angles of Δ ADB are equal to all angles of Δ CDA.

(i) $\Delta ADB \sim \Delta CDA$ **Proved**

(ii) $\frac{CD}{AD} = \frac{AD}{BD}$

$AD^2 = CD \times BD$

$= 8 \times 18 \Rightarrow AD = 12$ **Ans.**

(iii) $\frac{\Delta ADB}{\Delta CDA} = \frac{\frac{1}{2} AD \times BD}{\frac{1}{2} AD \times CD}$

$= \frac{BD}{CD} = \frac{18}{8}$

9

Ans.

Question 8.

(a) (i) *Using step-deviation method, calculate the mean marks of the following distribution.*

(ii) *State the modal class :* **[5]**

Class Interval	Frequency
50–55	5
55–60	20
60–65	10
65–70	10
70–75	9
75–80	6
80–85	12
85–90	8

(b) *Marks obtained by 200 students in an examination are given below :*

Marks	No. of Students
0–10	5
10–20	11
20–30	10
30–40	20
40–50	28
50–60	37
60–70	40
70–80	29
80–90	14
90–100	6

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis. Using the graph, determine

(i) *The median marks.*

(ii) *The number of students who failed if minimum marks required to pass is 40.*

(iii) *If scoring 85 and more marks is considered as grade one, find the number of students who secured grade on in the examination.* **[5]**

Solution :

(a) (i)

C.I.	f	x	$d = x - 67{\cdot}5$	u	fu
50–55	5	52·5	–15	–3	–15
55–60	20	57·5	–10	–2	–40
60–65	10	62·5	–5	–1	–10
65–70	10	67·5	0	0	0
70–75	9	72·5	5	1	9
75–80	6	77·5	10	2	12
80–85	12	82·5	15	3	36
85–90	8	87·5	20	4	32
	$\Sigma f = 80$				$\Sigma fu = 24$

A.M. = 67·5

$$\bar{x} = \text{A.M.} + \frac{\Sigma fu}{\Sigma f} \times i$$

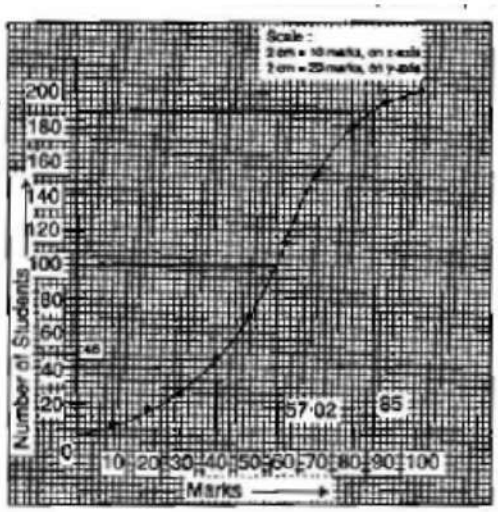
$$= 67{\cdot}5 + \frac{24}{80} \times 5$$

$$= 67{\cdot}5 + 1{\cdot}5 = 69$$ **Ans.**

(ii) Modal class is 55–60 (class with heighest freq.) **Ans.**

(b)

C.I.	f	C.F.
0–10	5	5
10–20	11	16
20–30	10	26
30–40	20	46
40–50	28	74
50–60	37	111
60–70	40	151
70–80	29	180
80–90	14	194
90–100	6	200



(i) $n = 200$

$$\text{Median} = \left(\frac{n}{2}\right)^{\text{th}} \text{observation}$$

$$= \left(\frac{200}{2}\right)^{\text{th}} \text{observation}$$

$$= 100^{\text{th}} \text{ observation} = 57{\cdot}02 \quad \textbf{Ans.}$$

(ii) Number of students who failed = 46 **Ans.**

(iii) Number of students who secured grade one = 200 – 1888 = 12. **Ans.**

Question 9.

(a) *Mr. Parekh invested ₹ 52,000 on 100 shares at a discount of ₹ 20 paying 8% dividend. At the end of one year he sells the shares at a premium of ₹ 20. Find*

(i) *The annual dividend.*

(ii) *The profit earned including his dividend.* **[3]**

(b) *Draw a circle of radius 3.5 cm. Mark a point P outside the circle at a distance of 6 cm from the centre. Construct two tangents from P to the given circle. Measure and write down the length of one tangent.* **[3]**

(c) *Prove that (cosec A – sin A) (sec A – cos A) $sec^2 A = tan A$.* **[4]**

Solution :

(a) Investment = ₹ 52,000, N.V = ₹ 100, M.V. of one share = ₹ (100 – 20) = ₹ 80, Dividend = 8%

$$\text{No. of shares} = \frac{\text{Investment}}{\text{MV}}$$

$$= \frac{52{,}000}{80}$$

$$= ₹\,650$$

(i) $$\text{Annual Dividend} = \frac{8}{100} \times 650 \times 100$$

$$= ₹\,5{,}200 \quad \textbf{Ans.}$$

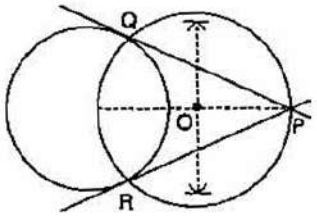
(ii) $\text{Profit} = \text{Total S.P.} + \text{Dividend} - \text{Investment}$

$= 650 \times 120 + 5{,}200 - 52{,}000$

$= 78{,}000 + 5{,}200 - 52{,}000$

$= ₹\,31{,}200$ **Ans.**

(b) Length of the tangent = 4·8 cm.



(c) $\text{L.H.S.} = (\operatorname{cosec} A - \sin A)(\sec A - \cos A) \cdot \sec^2 A$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right) \cdot \sec^2 A$

$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right) \cdot \sec^2 A$

$= \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} \times \frac{1}{\cos^2 A}$

$= \frac{\sin A}{\cos A}$

$= \tan A = \text{R.H.S.}$ **Hence Proved**

Question 10.

(a) *6 is the mean proportion between two numbers x and y and 48 is the third proportional of x and y. Find the numbers.* **[3]**

(b) *In what period of time will ₹ 12,000 yield ₹ 3,972 as compound interest at 10% per annum, if compounded on an yearly basis ?* **[3]**

(c) *A man observes the angle of elevation of the top of a building to be 30°. He walks towards it in a horizontal line through its base. On covering 60 m the angle of elevation changes to 60°. Find the height of the building correct to the nearest metre.* **[4]**

Solution :

(a) $xy = 6^2$

$\Rightarrow \quad xy = 36 \quad \ldots(1)$

$x : y :: y : 48$

$\frac{x}{y} = \frac{y}{48}$

$\Rightarrow \quad y^2 = 48x \quad \ldots(2)$

Subtituting the value of x from (1),

$$y^2 = 48 \times \frac{36}{y}$$

$\Rightarrow \quad y^3 = 48 \times 36$

$y^3 = 6 \times 8 \times 6 \times 6$

$y = 12$

$x = 3$ **Ans.**

(b) *Given* : P = ₹ 12,000, C.I. = ₹ 3,972, R% = 10% p.a.

Let $A = P\left(1 + \frac{R}{100}\right)^n$

$\Rightarrow \quad 15{,}972 = 12{,}000\left(1 + \frac{10}{100}\right)^n$

$\frac{1331}{1000} = \left(\frac{11}{10}\right)^n$

$\left(\frac{11}{10}\right)^3 = \left(\frac{11}{10}\right)^n$

$\Rightarrow \quad n = 3$ years. **Ans.**

(c) Let BC = x and AB = h

In right angled Δ ADB

$\tan 30° = \frac{h}{60 + x}$

$\Rightarrow \quad 60 + x = h\sqrt{3}$

Now right angled Δ ACB

$\tan 60° = \frac{h}{x}$

$x = \frac{h}{\sqrt{3}}$

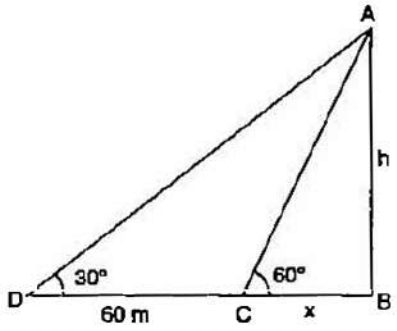
Equating 'x', $h\sqrt{3} - 60 = \frac{h}{\sqrt{3}}$

$\Rightarrow \quad 3h - 60\sqrt{3} = h$

$2h = 60\sqrt{3}$

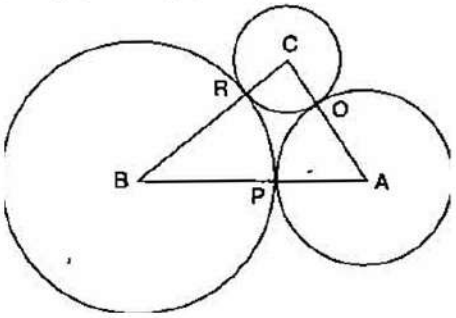
$h = 30\sqrt{3}$

$= 51{\cdot}96$ m. **Ans.**



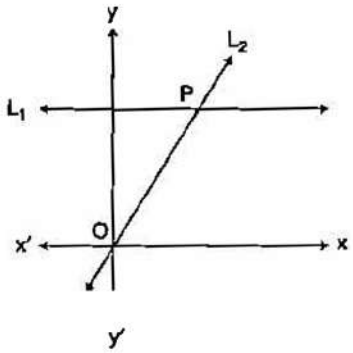
Question 11.

(a) *ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6 cm (not drawn to scale). Three circles are drawn touching each other with the vertices as their centres. Find the radii of the three circles.* **[3]**



(b) *₹ 480 is divided equally among 'x' children. If the number of children were 20 more then each would have got ₹ 12 less. Find 'x'.* **[3]**

(c) *Given equation of line L_1 is $y = 4$.*



(i) *Write the slope of line L_2 if L_2 is the bisector of angle O.*

(ii) *Write the co-ordinates of point P.*

(iii) *Find the equation of L_2.* **[4]**

Solution :

(a) Let the three radii be x, y, z respectively.

$$x + y = 10 \quad ...(1)$$
$$y + z = 8 \quad ...(2)$$
$$x + z = 6 \quad ...(3)$$

Adding equation's (1), (2) and (3), $2x + 2y + 2z = 24$

$$x + y + z = 12 \quad ...(4)$$

Subtracting each equation (1), (2) and (3) from equation (4), we get

$z = 2$ cm, $x = 4$ cm, $y = 6$ cm. **Ans.**

(b) Initial share of each child $= \frac{480}{x}$

New share of each child $= \frac{480}{x + 20}$

Difference in share is ₹ 12

$$\frac{480}{x} - \frac{480}{x+20} = 12$$
$$\frac{1}{x} - \frac{1}{x+20} = \frac{12}{480} = \frac{1}{40}$$
$$\frac{x + 20 - x}{x(x+20)} = \frac{1}{40}$$
$$x^2 + 20x = 800$$
$$x^2 + 20x - 800 = 0$$
$$x^2 + 40x - 20x - 800 = 0$$
$$x(x + 40) - 20(x + 40) = 0 \text{ (not possible)}$$
$$x = 20 \text{ or } x = -40$$
$$x = 20$$ **Ans.**

(c) (i). Slope of L_2 is $\quad m = \tan 45°$

$m = 1$ (L_2 makes an angle of 45° with X axis)

(ii) Equation of line L_2

$y - 0 = 1(x - 0)$ It passes through (0, 0)

$\therefore$ Equation of L_2 is $\quad y = x$

P can be obtained by solving L_1 and L_2 simultaneously,

$L_1 \quad y = 3$

$L_2 \quad y = x$

On solving, we get $\quad x = 3, y = 3$

Co-ordinate of P (3, 3) **Ans.**

(iii) Equation of L_2 is $y = x$ [as solved above part (ii)]. **Ans**

●●

ICSE Question Paper (2012)

MATHEMATICS

SECTION A [40 Marks]

*(Answer **all** questions from this Section.)*

Question 1.

(a) *If* $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ *and* $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, *find* $A^2 - 5A + 7I$. **[3]**

(b) *The monthly pocket money of Ravi and Sanjeev are in the ratio 5 : 7. Their expenditures are in the ratio 3 : 5. If each saves ₹ 80 every month, find their monthly pocket money.* **[3]**

(c) *Using the Remainder Theorem factorise completely the following polynomial :*

$$3x^3 + 2x^2 - 19x + 6$$ **[4]**

Solution :

(a) Let $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

than $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+10 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$ **Ans.**

(b) Let monthly pocket money of Ravi is $5x$ and Sanjeev is $7x$.

$$\frac{5x - 80}{7x - 80} = \frac{3}{5}$$

$$\Rightarrow \quad 25x - 400 = 21x - 240$$

$$4x = 160$$

$$x = 40$$

Ravi's pocket money $= 5 \times 40 = ₹ 200$

Sanjeev's pocket money $= 7 \times 40 = ₹ 280$ **Ans.**

(c) Let $f(x) = 3x^3 + 2x^2 - 19x + 6$

Using hit and trial method,

$$f(1) = 3 + 2 - 19 + 6 \neq 0$$

$f(-1) = -3 + 2 + 19 + 6 \neq 0$

$f(2) = 24 + 8 - 38 + 6 = 0$

$\therefore (x - 2)$ is a factor of $f(x)$.

Now,

$$\begin{array}{r} 3x^2 + 8x - 3 \\ x - 2)\ 3x^3 + 2x^2 - 19x + 6 \\ 3x^3 - 6x^2 \\ \hline 8x^2 - 19x \\ 8x^2 - 16x \\ \hline -3x + 6 \\ -3x + 6 \\ \hline \times \end{array}$$

To factorise $3x^2 + 8x - 3$

$= 3x^2 + 9x - x - 3$

$= 3x(x + 3) - 1(x + 3)$

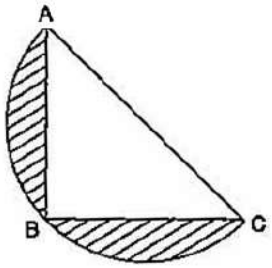
$= (3x - 1)(x + 3)$

Hence $3x^3 + 2x^2 - 19x + 6 = (x - 2)(3x - 1)(x + 3)$ **Ans.**

Question 2.

(a) *On what sum of money will the difference between the compound interest and simple interest for 2 years be equal to ₹ 25 if the rate of interest charged for both is 5% p.a. ?* **[3]**

(b) *ABC is an isosceles right angled triangle with ∠ABC = 90°. A semi-circle is drawn with AC as the diameter. If AB = BC = 7 cm, find the area of the shaded region.* $\left(\text{Take } \pi = \frac{22}{7}\right)$ **[3]**



(c) *Given a line segment AB joining the points A (–4, 6) and B (8, –3). Find :*

(i) *the ratio in which AB is divided by the y-axis.*

(ii) *find the coordinates of the point of intersection.*

(iii) *the length of AB.* **[4]**

Solution :

(a) Let the principal be ₹ P.

Given : R = 5%, T = 24 years

$$\text{C.I. for 2 years} = P\left(1 + \frac{5}{100}\right)^2 - P$$

$$\text{S.I. for 2 years} = \frac{P \times 5 \times 2}{100} = \frac{P}{10}$$

$\because$ Difference between C.I. and S.P. = ₹ 25

$$P\left(1 + \frac{5}{100}\right)^2 - P - \frac{P}{10} = 25$$

$$\frac{441\,P}{400} - \frac{11P}{10} = 25$$

$$\frac{441P - 440P}{400} = 25$$

$$P = 10{,}000$$

Hence, the principle be 10,000 **Ans.**

(b) Let ABC is a right angled triangle. So

$$AC^2 = AB^2 + BC^2$$

$$= (7)^2 + (7)^2 = 2(7)^2$$

$$AC = 7\sqrt{2}$$

$$\text{Area of semi circle} = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7\sqrt{2}}{2}\right)^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times \frac{49 \times 2}{4}$$

$$= 38{\cdot}5 \text{ cm}^2$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times 7 \times 7 = 24{\cdot}5 \text{ cm}^2$$

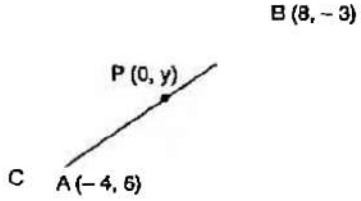
$$\text{Area of shaded region} = \text{Area of semi circle} - \text{Area of } \Delta ABC.$$

$$= 38{\cdot}5 - 24{\cdot}5 = 14 \text{ cm}^2. \quad \textbf{Ans.}$$

(c) Let P be the point at which

(i) AB intersect y-axis

Let $AP : PB = m : n$



$$x = \frac{mx_1 + nx_2}{m+n}$$

and $$y = \frac{my_1 + ny_2}{m+n}$$

$$0 = \frac{m \cdot 8 + n(-4)}{m+n}$$

$$\frac{8m - 4n}{m+n} = 0$$

$$8m = 4n$$

$$m : n = 1 : 2 \quad \textbf{Ans.}$$

(ii) $$y = \frac{my_2 + ny_1}{m+n}$$

$$\frac{1 \times (-3) + 2 \times 6}{1+2}$$

Using the above ratio, $$y = \frac{-3+12}{1+2} = 3$$

$\therefore$ Point of P be (0, 3) **Ans.**

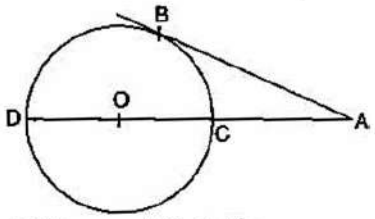
(iii) $$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-4-8)^2 + (6+3)^2}$$

$$= \sqrt{144 + 81} = 15 \text{ units.} \quad \textbf{Ans.}$$

Question 3.

(a) *In the given figure O is the centre of the circle and AB is a tangent at B. If AB = 15 cm and AC = 7.5 cm. Calculate the radius of the circle.* **[3]**



(b) *Evaluate without using trigonometric tables :*

$$\cos^2 26° + \cos 64° \sin 26° + \frac{\tan 36°}{\cot 54°}$$ **[3]**

(c) *Marks obtained by 40 students in a short assessment is given below, where a and b are two missing data.*

Marks	*5*	*6*	*7*	*8*	*9*
No. of students	*6*	*a*	*16*	*13*	*b*

If the mean of the distribution is 7.2, find a and b. **[4]**

Solution :

(a) Applying intercept theorem

$$AC \times AD = AB^2$$
$$7{\cdot}5 \times (7{\cdot}5 + 2R) = 15^2$$

where R is the radius of the circle

$$(7{\cdot}5 + 2R) = \frac{15 \times 15}{7{\cdot}5} = 30$$
$$2R = 22{\cdot}5$$
$$R = 11{\cdot}25 \text{ cm.}$$ **Ans.**

(b) *Given :* $\cos^2 26° + \cos 64° \sin 26° + \frac{\tan 36°}{\cot 54°}$

$$= \cos^2 26° + \cos (90° - 26°) \cdot \sin 26° + \frac{\tan (90° - 54°)}{\cot 54°}$$

$$= (\cos^2 26° + \sin^2 26°) + \frac{\cot 54°}{\cot 54°}$$

$$= 1 + 1 = 2$$ **Ans.**

(c) Let $6 + a + 16 + 13 + b = 40$

$\Rightarrow$ $a + b = 5$ ...(i)

Mean $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$$7{\cdot}2 = \frac{30 + 6a + 112 + 104 + 9b}{40}$$
$$246 + 6a + 9b = 288$$
$$6a + 9b = 42$$
$$2a + 3b = 14$$ ...(ii)

Solving (i) and (ii), we get

$$b = 4, \ a = 1$$ **Ans.**

Question 4.

(a) *Kiran deposited ₹ 200 per month for 36 months in a bank's recurring deposit account. If the bank pays interest at the rate of 11% per annum, find the amount she gets on maturity.* **[3]**

(b) *Two coins are tossed once. Find the probability of getting :*

(i) *2 heads*

(ii) *at least 1 tail.* **[3]**

(c) *Using graph paper and taking 1 cm = 1 unit along both x-axis and y-axis.*

(i) *Plot the points A(–4, 4) and B (2, 2)*

(ii) *Reflect A and B in the origin to get the images A′ and B′ respectively.*

(iii) *Write down the co-ordinates of A′ and B′.*

(iv) *Give the geometrical name for the figure ABA′B′.*

(v) *Draw and name its lines of symmetry.* **[4]**

Solution :

(a) *Given* : P per month = ₹ 200, Time (n) = 36 months, R = 11%.

$$\text{Equivalent principal for 36 months} = 200 \times \frac{n(n+1)}{2}$$

$$= 200 \times \frac{36 \times 37}{2}$$

$$= 36 \times 37 \times 100$$

$$\text{Interest} = \frac{PRT}{100}$$

$$= \frac{36 \times 37 \times 100 \times 11 \times 1}{100 \times 12}$$

$$= ₹\,1221$$

$$\text{Maturity Amount} = Pn + \text{Interest}$$

$$= 200 \times 36 + 1221$$

$$= ₹\,8421. \quad \textbf{Ans.}$$

(b) If two coins are tossed once, then

$$\text{Sample Space (S)} = \{H\,H, HT, TH, TT\}$$

$$n(S) = 4$$

(i) $$E : \text{getting two heads} = \{H\,H\}$$

$$n(E) = 1$$

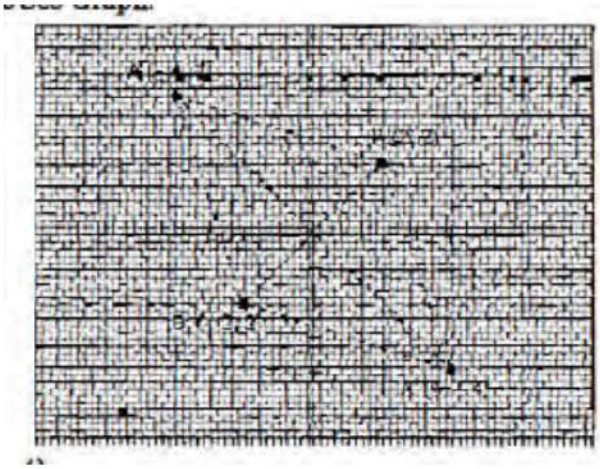
$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{4} \quad \textbf{Ans.}$$

(ii) $$E : \text{At least one tail} = \{HT, TH, TT\}$$

$$n(E') = 3$$

$$P(E') = \frac{n(E')}{n(S)} = \frac{3}{4} \quad \textbf{Ans.}$$

(c) (i) Please See Graph.
(ii) Please See Graph.



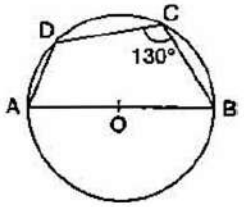
(iii) A′ (4, –4)
B′ (–2, –2)
(iv) Rhombus
(v) Two lines of symmetry.
Both diagonals, AA′ and BB′

SECTION B [40 Marks]

*Answer any **four** Questions in this Section.*

Question 5.

(a) *In the given figure, AB is the diameter of a circle with centre O.*
∠BCD = 130°. Find :
(i) *∠DAB*, (ii) *∠DBA* **[3]**



(b) *Given* $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$. *Write :*
(i) *the order of the matrix X*
(ii) *the matrix X.* **[3]**

(c) *A page from the Savings Bank Account of Mr. Prateek is given below :*

Date	*Particulars*	*Withdrawal (In ₹)*	*Deposit (In ₹)*	*Balances (In ₹)*
Jan. 1st 2006	*B/F*	—	—	*1,270*
Jan. 7th 2006	*By Cheque*	—	*2,310*	*3,580*
March 9th 2006	*To Self*	*2,000*	—	*1,580*
March 26th 2006	*By Cash*	—	*6,200*	*7,780*
June 10th 2006	*To Cheque*	*4,500*	—	*3,280*
July 15th 2006	*By Clearing*	—	*2,630*	*5,910*
October 18th 2006	*To Cheque*	*530*	—	*5,380*
October 27th 2006	*To Self*	*2,690*	—	*2,690*
November 3rd 2006	*By Cash*	—	*1,500*	*4,190*
December 6th 2006	*To Cheque*	*950*	—	*3,240*
December 23rd 2006	*By Transfer*	—	*2,920*	*6,160*

If he receives ₹ 198 as interest on 1st January, 2007, find the rate of interest paid by the bank. **[4]**

Solution :

(a) On joining BD, ∠ ADB is in the semicircle.

$$\angle ADB = 90°$$

(Angle in a simicircle is right angle)

(i) Let ABCD is a cyclic quadilateral.

$$\angle BCD + \angle DAB = 180°$$
$$130° + \angle DAB = 180°$$
$$\angle DAB = 180° - 130° = 50°$$ **Ans.**

(ii) Now, $\angle BAD + \angle BDA + \angle DBA = 180°$

$$90° + 50° + \angle DBA = 180°$$
$$\angle DBA = 40°$$ **Ans.**

(b) (i) Order of matrix X is 2×1.

(ii) Let $X = \begin{bmatrix} a \\ b \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$
$$\begin{bmatrix} 2a+b \\ -3a+4b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$
$$2a + b = 7 \quad \text{.....(1)}$$
$$-3a + 4b = 6 \quad \text{.....(2)}$$

Solving (1) and (2), we get

$$a = 2,\ b = 3$$
$$X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$ **Ans.**

(c)

Months	Minimum Balance
January	3,580
February	3,580
March	1,580
April	7,780
May	7,780
June	3,280
July	3,280
August	5,910
September	5,910
October	2,690
November	4,190
December	3,240
Total	₹ 52,800

Now, Principal = ₹ 52,800

Time = 1 month = $\frac{1}{12}$ year,

Interest = ₹ 198

$I = \frac{PRT}{100}$

$\frac{52,800 \times R \times 1}{100 \times 12} = 198$

R = 4·5%. **Ans.**

Question 6.

(a) *The printed price of an article is ₹ 60,000. The wholesaler allows a discount of 20% to the shopkeeper. The shopkeeper sells the article to the customer at the printed price. Sales tax (under VAT) is charged at the rate of 6% at every stage. Find :*

(i) *the cost to the shopkeeper inclusive of tax.*

(ii) *VAT paid by the shopkeeper to the Government.*

(iii) *the cost to the customer inclusive of tax.* **[3]**

(b) *Solve the following inequation and represent the solution set on the number line :*

$$4x - 19 < \frac{3x}{5} - 2 \leq \frac{-2}{5} + x, \; x \in R$$ **[3]**

(c) *Without solving the following quadratic equation, find the value of 'm' for which the given equation has real and equal roots.*

$$x^2 + 2(m-1)x + (m+5) = 0$$ **[3]**

Solution :

(a) (i) *Given :* Printed price of the article = ₹ 60,000

and discount = 20% of ₹ 60,000

$= \frac{20}{100} \times 60,000 =$ ₹ 12,000

Sale price of the article = 60,000 – 12,000 = ₹ 48,000

Sales tax paid by the shopkeeper = 6% of ₹ 48,000

$= \frac{6}{100} \times 48,000 =$ ₹ 2,880

∵ The cost of the shopkeeper inclusive of tax

= 48,000 + 2,880

= ₹ 50,880. **Ans.**

(ii) VAT paid by shopkeeper = Tax charged – Tax paid

$= 60,000 \times \frac{6}{100} - 48,000 \times \frac{6}{100}$

= ₹ 720 **Ans.**

(iii) Sales tax paid by the customer = 6% of ₹ 60,000

$= \frac{6}{100} \times 60,000 =$ ₹ 3,600

∵ The cost to the customer inclusive of tax :

= 60,000 + 3,600

= ₹ 63,600. **Ans.**

(b) *Given* : $4x - 19 < \frac{3x}{5} - 2 \leq -\frac{2}{5} + x$

$$4x - 19 < \frac{3x}{5} - 2 \quad \text{and} \quad \frac{3x}{5} - 2 \leq -\frac{2}{5} + x$$

$$\frac{17x}{5} < 17 \quad \text{and} \quad -\frac{2x}{5} \leq \frac{8}{5}$$

$$x < 5 \quad \Rightarrow \quad x \geq -4$$

Solution set $= \{x : 5 > x \geq -4\}$

$-4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6$

(c) *Given :* $x^2 + 2(m-1)x + (m+5) = 0$ ...(i)

For real and equal roots,

$$b^2 - 4ac = 0$$

$$b^2 = 4ac$$

Comparing given equation (i) with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 2(m-1), c = (m+5)$$

Now, $4(m-1)^2 = 4(m+5)$

$$m^2 - 3m - 4 = 0$$

$$m^2 - 4m + m - 4 = 0$$

$$m(m-4) + 1(m-4) = 0$$

$$m = 4 \text{ or } m = -1$$ **Ans.**

Question 7.

(a) *A hollow sphere of internal and external radii 6 cm and 8 cm respectively is melted and recast into small cones of base radius 2 cm and height 8 cm. Find the number of cones.* **[3]**

(b) *Solve the following equation and give your answer correct to 3 significant figures :*

$$5x^2 - 3x - 4 = 0$$ **[3]**

(c) *As observed from the top of a 80 m tall lighthouse, the angles of depression of two ships on the same side of the light house in horizontal line with its base are 30° and 40° respectively. Find the distance between the two ships. Give your answer correct to the nearest metre.* **[4]**

Solution :

(a) *Given* : External Radius R = 8 cm, Internal Radius = 6 cm,

$$\text{Volume of hollow spheres} = \frac{4}{3}\pi(R^3 - r^3).$$

$$\text{Volume of hollow spheres} = \frac{4}{3}\pi[8^3 - 6^3]$$

$$= \frac{4}{3}\pi[512 - 216] = \frac{4}{3}\pi(296)$$

$$\text{Volume of cones} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(2)^2(8)$$

$$\text{Number of cones} = \frac{\text{Volume of sphere}}{\text{Volume of cones}} = \frac{\frac{4}{3}\pi\,[296]}{\frac{1}{3}\pi \times 4 \times 8}$$

$$= \frac{296}{8} = 37 \text{ cones.} \qquad \textbf{Ans.}$$

(b) *Given* : $5x^2 - 3x - 4 = 0$

Comparing given equation with $ax^2 + bx + c = 0$, we get

$$a = 5, b = -3, c = -4$$

Let $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 5 \times (-4)}}{2 \times 5}$$

$$x = \frac{3 \pm \sqrt{9 + 80}}{10} = \frac{3 \pm \sqrt{89}}{10} = \frac{3 \pm 9{\cdot}43}{10}$$

Taking *+ve* sign

$$x = \frac{3 + 9{\cdot}43}{10} = 1{\cdot}243$$

and taking *– ve* sign

$$x = \frac{3 - 9{\cdot}43}{10}$$

$$= \frac{-6{\cdot}43}{10}$$

$$= -0{\cdot}643$$

$x = 1{\cdot}243$ or $x = -0{\cdot}643$ **Ans.**

(c) In Δ ABC, $\tan 50° = \frac{BC}{80}$

$BC = 80 \times 1{\cdot}1918$

$BC = 95{\cdot}34$ m

In Δ ABD, $\tan 60° = \frac{BD}{80}$

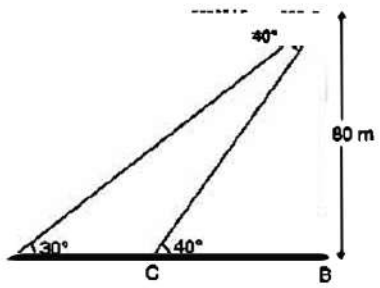
$BD = 80\sqrt{3}$

$BD = 138{\cdot}56$ m

$CD = BD - BC$

$= 138{\cdot}56 - 95{\cdot}34$

$= 43{\cdot}2$ m. **Ans.**



Question 8.

(a) *A man invests ₹ 9,600 on ₹ 100 shares at ₹ 80. If the company pays him 18% dividend find*

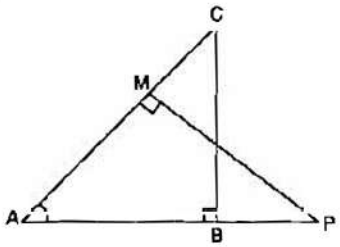
(i) *the number of shares he buys.*

(ii) *his total dividend.*

(iii) *his percentage return on the shares.* **[3]**

(b) *In the given figure Δ ABC and Δ AMP are right angled at B and M respectively.*

Given AB = 10 cm, AP = 15 cm and PM = 12 cm.



(i) *Prove Δ ABC ~ Δ AMP.*

(ii) *Find AB and BC.* **[3]**

(c) *If* $x = \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$, *using properties of proportion show that* $x^2 - 2ax + 1 = 0$. **[4]**

Solution :

(a) *Given* : Investment = ₹ 9,600, N.V. = ₹ 100, M.V. = ₹ 80, Div. % = 18%

(i) $\text{Number of shares} = \frac{\text{Investment}}{\text{M.V. of each share}}$

$= \frac{9600}{80} = 120 \text{ shares}$ **Ans.**

(ii) $\text{Total dividend} = \frac{18}{100} \times 120 \times 100$

$= ₹ 2{,}160$ **Ans.**

(iii) Since $\text{N.V} \times \text{Div\%} = \text{M.V.} \times \text{Return \%}$

$\text{Return \%} = \frac{100 \times 189}{80}$

$= 22{\cdot}5\%$ **Ans.**

(b) (i) In Δ ABC and Δ APM,

$\angle ABC = \angle AMP = 90°$

$\angle BAC = \angle PAM$ (Common)

$\Delta ABC \sim \Delta APM$

(ii) Also, $\frac{AC}{AP} = \frac{BC}{PM}$

$\frac{10}{15} = \frac{BC}{12}$

$BC = 8 \text{ cm.}$ **Ans.**

∵ Δ ABC is right angled Δ.

Applying Pythagorous,

$AB^2 = AC^2 - BC^2$

$= 10^2 - 8^2$

$AB = 6 \text{ cm.}$ **Ans.**

(c) *Given* : $\frac{x}{1} = \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$

Using componendo and dividendo,

$$\frac{x+1}{x-1} = \frac{\sqrt{a+1}}{\sqrt{a-1}}$$

Squaring both sides, $\frac{(x+1)^2}{(x-1)^2} = \frac{a+1}{a-1}$

again using componendo and dividendo,

$$\frac{x^2+1}{2x} = \frac{a}{1}.$$

$$x^2 - 2ax + 1 = 0$$ **Hence Proved**

Question 9.

(a) *The line through A (–2, 3) and B (4, b) is perpendicular to the line 2x – 4y = 5. Find the value of b.* **[3]**

(b) *Prove that* $\frac{\tan^2\theta}{(\sec\theta - 1)^2} = \frac{1+\cos\theta}{1-\cos\theta}$ **[3]**

(c) *A car covers a distance of 400 km at a certain speed. Had the speed been 12 km/h more, the time taken for the journey would have been 1 hour 40 minutes less. Find the original speed of the car.* **[4]**

Solution :

(a) Slope of AB $(m_1) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b-3}{4+2} = \frac{b-3}{6}$

Equation of given line $2x - 4y = 5$

$$4y = 2x - 5$$

$$y = \frac{1}{2}x - \frac{5}{4}$$

Slope of given line $(m_2) = \frac{1}{2}$

As per the question, line are perpendicular.

$$m_1 \cdot m_2 = -1$$

$$\frac{b-3}{6} \times \frac{1}{2} = -1$$

$\Rightarrow$ $b - 3 = -12$

$b = -9$ **Ans.**

(b) L.H.S. $= \frac{\tan^2\theta}{(\sec\theta - 1)^2}$

$$= \frac{\sec^2\theta - 1}{(\sec\theta - 1)^2}$$

$$= \frac{(\sec\theta - 1)(\sec\theta + 1)}{(\sec\theta - 1)^2}$$

$$= \frac{\sec\theta + 1}{\sec\theta - 1} \quad \frac{\frac{1}{\cos\theta} + 1}{\frac{1}{\cos\theta} - 1}$$

$$= \frac{1+\cos\theta}{1-\cos\theta} = \text{R.H.S.}$$ **Hence Proved**

(c) Let the original speed of car be x km/hr.

Usual time $= \frac{400}{x}$, New speed $= x + 12$, New time $= \frac{400}{x+1}$

According to the condition :

$$\frac{400}{x} - \frac{400}{x+12} = \frac{5}{3}$$

$$\frac{x+12-x}{x(x+12)} = \frac{1}{240}$$

$$x^2 + 12x - 2880 = 0$$

$$x^2 + 60x - 48x - 2880 = 0$$

$$x(x+60) - 48(x+60) = 0$$

$$x = -60 \text{ or } x = 48$$

But speed can not be negative.

Original speed = 48 km/hr **Ans.**

Question 10.

(a) *Construct a triangle ABC in which base BC = 6 cm, AB = 5.5 cm and ∠ABC = 120°.*

(i) *Construct a circle circumscribing the triangle ABC.*

(ii) *Draw a cyclic quadrilateral ABCD so that D is equidistant from B and C.*

[3]

(b) *The following distribution represents the height of 160 students of a school.*

Height (in cm)	*No. of students*
140-145	*12*
145-150	*20*
150-155	*30*
155-160	*38*
160-165	*24*
165-170	*16*
170-175	*12*
175-180	*8*

Draw an ogive for the given distribution taking 2 cm = 5 cm of height on one axis and 2 cm = 20 students on the other axis. Using the graph, determine :

(i) *The median height.*

(ii) *The interquartile range.*

(iii) *The number of students whose height is above 172 cm.* **[6]**

Solution :

(a) (i) **Steps of constructions :**

(1) Draw a line segment BC = 6 cm.

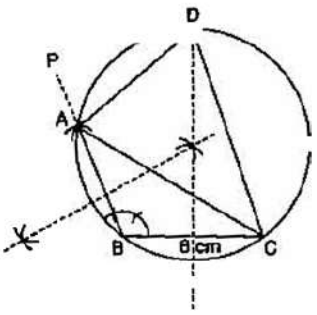
(2) Construct ∠CBP = 120°.

(3) Cut BA = 5·5 cm from BP.

(4) Join A to C.

(5) Construct perpendicular bisectors of AB and BC, intersecting at O. Join AO.

(6) Taking as the centre and OA as radius draw a circle, passing through, A, B, and C.



(ii) (1) Extend the right bisector of BC intersecting the circle at D.

(2) Join A to D and C to D.

(3) ABCD is required cyclic quadrilateral.

(b)

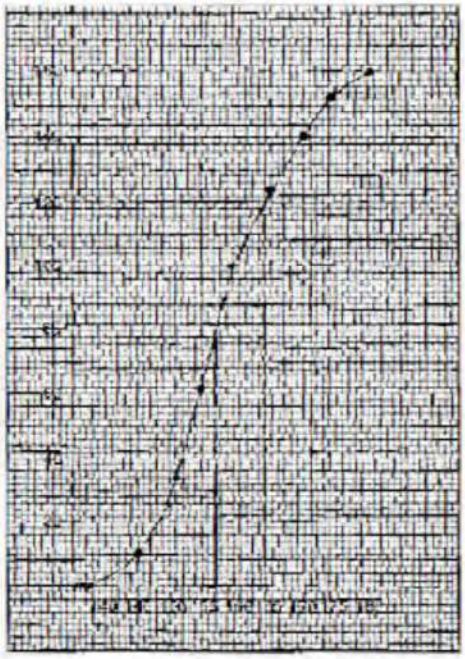
Height	f	$c.f.$
140–145	12	12
145–150	20	32
150–155	30	62
155-160	38	100
160-165	24	124
165-170	16	140
170-175	12	152
175-180	8	160

(i) Me = 157·3 **Ans.**

(ii) Interquartile range

$= Q_3 - Q_1$

$= 164{\cdot}1 - 151{\cdot}3 = 12{\cdot}8$ **Ans.**

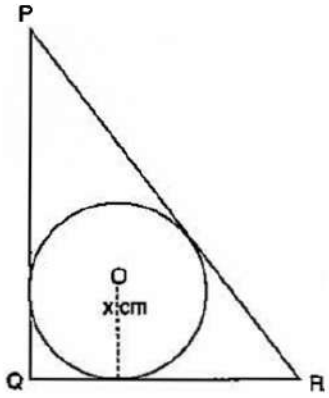


(iii) No. of students above 172 cm = 160 – 144 = 16. **Ans.**

Question 11.

(a) *In triangle PQR, PQ = 24 cm, QR = 7 cm and ∠ PQR = 90°.*

Find the radius of the inscribed circle. **[3]**



(b) *Find the mode and median of the following frequency distribution :* **[3]**

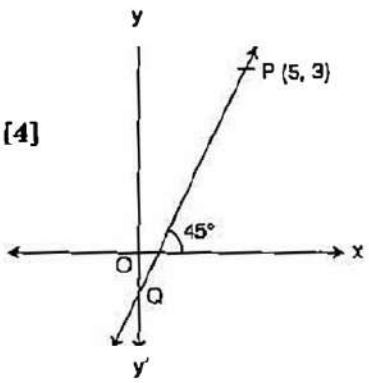
x	10	11	12	13	14	15
f	1	4	7	5	9	3

(c) *The line through P(5, 3) intersects y axis at Q.*

(i) *Write the slope of the line.*

(ii) *Write the equation of the line.*

(iii) *Find the co-ordinates of Q.* **[4]**

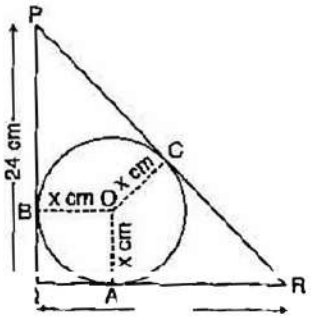


Solution :

(a) *Given* : Δ PQR is right angled.

$$PR^2 = PQ^2 + QR^2$$
$$= (24)^2 + (7)^2$$
$$= 576 + 49 = 625$$
$$PR = 25 \text{ cm}$$

Draw ⊥r from O on PQ and PR and mark as B and C respectively.



$$\angle OBQ = \angle OAQ = \angle OCR$$
$$= 90°$$

(∠ between radius and tangent is 90°)

All ∠'s of OAQB are 90° and QA = QB

(Since the tangent to a circle from an exterior point are equal in length).

∴ OAQB is a square.

$$QA = QB = x$$
$$AR = 7 - x = RC \quad (\because AR = RC)$$
$$BP = 12 - x = PC \quad (\because PB = PC)$$

Tangents from ext. point are equal

$$PC + RC = PR$$
$$7 - x + 12 - x = 25$$
$$x = 3 \text{ cm}$$ **Ans.**

(b) Mode is the value of the highest frequency.

Mode = 14 **Ans.**

For Median, we write the data in ascending order,

10, 11, 11, 11, 11, 12, 12, 12, 12, 12, 12, 12, 13, 13, 13, 13, 13, 14, 14, 14, 14, 14, 14, 14, 14, 14, 15, 15, 15.

∵ Median is the middle most value.

$$M_e = \left(\frac{N+1}{2}\right)^{th} \text{ observation}$$
$$= \left(\frac{29+1}{2}\right)^{th} = 15\text{th observation} = 13$$ **Ans.**

(c) (i) Slope of line PQ = $\tan 45° = 1$ **Ans.**

(ii) Equation of line PQ :

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 1(x - 5)$$
$$y = x - 2$$ **Ans.**

(iii) Put $x = 0$ in equation of line PQ.

$$y = -2$$

Coordinate of Q = (0, –2) **Ans**

ICSE Question Paper (2013)

MATHEMATICS

SECTION A [40 Marks]

*(Answer **all** questions from this Section.)*

Question 1.

(a) *Given* $A=\begin{bmatrix}2 & -6\\ 2 & 0\end{bmatrix}, B=\begin{bmatrix}-3 & 2\\ 4 & 0\end{bmatrix}, C=\begin{bmatrix}4 & 0\\ 0 & 2\end{bmatrix}$.

Find the matrix X such that $A + 2X = 2B + C$. **[3]**

(b) *At what rate % p.a. will a sum of* ₹ *4000 yield* ₹ *1324 as compound interest in 3 years ?* **[3]**

(c) *The median of the following observations 11, 12, 14, (x – 2), (x + 4), (x + 9), 32, 38, 47 arranged in ascending order is 24. Find the value of x and hence find the mean.* **[4]**

Solution :

(a) *Given :* $A=\begin{bmatrix}2 & -6\\ 2 & 0\end{bmatrix}, B=\begin{bmatrix}-3 & 2\\ 4 & 0\end{bmatrix}$ and $C=\begin{bmatrix}4 & 0\\ 0 & 2\end{bmatrix}$

$$A + 2X = 2B + C$$

Putting the given values, we get

$$\begin{bmatrix}2 & -6\\ 2 & 0\end{bmatrix} + 2X = 2\begin{bmatrix}-3 & 2\\ 4 & 0\end{bmatrix} + \begin{bmatrix}4 & 0\\ 0 & 2\end{bmatrix}$$

$$2X = \begin{bmatrix}-6+4 & 4+0\\ 8+0 & 0+2\end{bmatrix} - \begin{bmatrix}2 & -6\\ 2 & 0\end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix}-4 & 10\\ 6 & 2\end{bmatrix}$$

$$X = \begin{bmatrix}-2 & 5\\ 3 & 1\end{bmatrix}$$ **Ans.**

(b) *Given :* Principal = ₹ 4,000, C.I. = ₹ 1,324,

Amount = P + C.I.

= ₹ (4,000 + 1,324) = ₹ 5,324

Time = 3 years

We know that, $A = P\left(1 + \frac{r}{100}\right)^T$

$$5{,}324 = 4{,}000\left(1 + \frac{r}{100}\right)^3$$

$$\frac{5{,}324}{4{,}000} = \left(1 + \frac{r}{100}\right)^3$$

$$\frac{1,331}{1,000} = \left(1 + \frac{r}{100}\right)^3$$

$$\left(\frac{11}{10}\right)^3 = \left(1 + \frac{r}{100}\right)^3$$

Therefore, $1 + \frac{r}{100} = \frac{11}{10}$

$$\frac{r}{100} = \frac{11}{10} - 1$$

$$\frac{r}{100} = \frac{1}{10}$$

$$r = \frac{100}{10}$$

$$r = 10\%$$ **Ans.**

(c) Given observation are 11, 12, 14, $(x - 2)$, $(x + 4)$, $(x + 9)$, 32, 38, 47 and median = 24.

$$n = 9 \text{ (odd)}$$

$$\text{Median} = \frac{n+1}{2} \text{ th term}$$

$$= \frac{9+1}{2} \text{ th term}$$

$$24 = 5\text{th term}$$

$$x + 4 = 24$$

$$x = 24 - 4$$

$$x = 20$$

Therefore, 11, 12, 14, (20 – 2), (20 + 4), (20 + 9), 32, 38, 47

= 11, 12, 14, 18, 24, 29, 32, 38, 47

Now $\text{Mean} = \frac{\Sigma x}{n}$

$$= \frac{11 + 12 + 14 + 18 + 24 + 29 + 32 + 38 + 47}{9}$$

$$= \frac{225}{9} = 25$$ **Ans.**

Question 2.

(a) *What number must be added to each of the number 6, 15, 20 and 43 to make them proportional ?* **[3]**

(b) *If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$, it leaves a remainder 52, find the values of a and b.* **[3]**

(c) *Draw a histogram from the following frequency distribution and find the mode from the graph :* **[4]**

Class	*0–5*	*5–10*	*10–15*	*15–20*	*20–25*	*25–30*
Frequency	*2*	*5*	*18*	*14*	*8*	*5*

Solution :

(a) Let the number must be added be x, then

$$\text{the new number} = 6+x,\ 15+x,\ 20+x,\ 43+x$$

$\because$ These are proportionals.

$$6+x : 15+x :: 20+x : 43+x$$

or $$(6+x)(43+x) = (15+x)(20+x)$$

or $$258 + 6x + 43x + x^2 = 300 + 20x + 15x + x^2$$

or $$49x - 35x = 300 - 258$$

or $$14x = 42$$

or $$x = 3.$$ **Ans.**

(b) Let $(x-2)$ is a factor of the given expression.

$$x - 2 = 0$$
$$x = 2$$

Given expression,

$$2x^3 + ax^2 + bx - 14 = 0$$
$$2(2)^3 + a(2)^2 + b(2) - 14 = 0$$
$$16 + 4a + 2b - 14 = 0$$
$$4a + 2b + 2 = 0$$
$$4a + 2b = -2$$
$$2a + b = -1 \quad \text{...(i)}$$

and when given expression is divided by $(x-3)$

$$x - 3 = 0$$
$\Rightarrow$ $$x = 3$$
$$2x^3 + ax^2 + bx - 14 = 52$$
$$2(3)^3 + a(3)^2 + b(3) - 66 = 0$$
$$54 + 9a + 3b - 66 = 0$$
$$9a + 3b = 12$$
$$3a + b = 4 \quad \text{...(ii)}$$

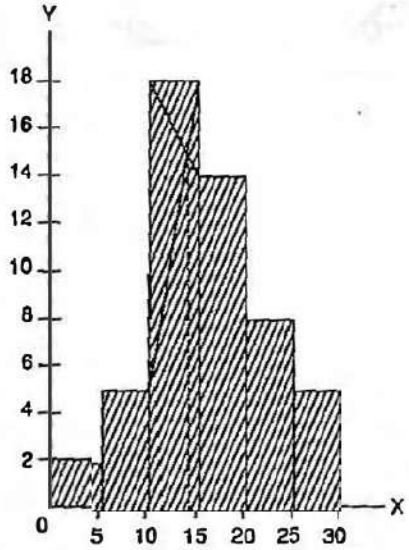
Solving equation (i) and (ii),

$$2a + b = -1$$
$$3a + b = 4$$
$$(-)\quad (-)\quad (+)$$
$$-a = -5$$
$$a = 5$$

from (ii),

$$3 \times 5 + b = 4$$
$$b = 4 - 15$$
$$b = -11$$
$$a = 5 \text{ and } b = -11$$ **Ans.**

(c)



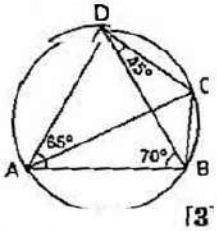
From the Histogram the value of Mode is **13·8.** **Ans.**

Question 3.

(a) *Without using tables evaluate 3 cos 80°.Cosec 10° + 2 sin 59° sec 31°.* **[3]**

(b) *In the given figure,*

$\angle BAD = 65°$,
$\angle ABD = 70°$,
$\angle BDC = 45°$



(i) *Prove that AC is a diameter of the circle.*

(ii) *Find* $\angle ACB$. **[3]**

(c) *AB is a diameter of a circle with centre C = (–2, 5). If A = (3, –7). Find :*

(i) *The length of radius AC*

(ii) *The coordinates of B.* **[4]**

Solution :

(a) *Given* :

$3 \cos 80° \cdot \text{cosec } 10° + 2 \sin 59° \sec 31°$

$= 3 \cos 80° \text{ cosec } (90° - 80°) + 2 \sin 59° \sec (90° - 59°)$

$= 3 \cos 80° \sec 80° + 2 \sin 59° \text{ cosec } 59°$

$= 3 \cos 80° \times \frac{1}{\cos 80°} + 2 \sin 59° \times \frac{1}{\sin 59°}$

$= 3 + 2 = 5.$ **Ans.**

(b) *Given* : $\angle BAD = 65°$, $\angle ABD = 70°$, $\angle BDC = 45°$

(i) $\because$ ABCD is a cyclic quadrilateral.

In ΔABD,

$\angle BDA + \angle DAB + \angle ABD = 180°$ By using sum property of Δ^s

$\therefore \quad \angle BDA = 180° - (65° + 70°)$

$= 180° - 135°$

$= 45°$

Now from Δ ACD,

$$\angle ADC = \angle ADB + \angle BDC$$
$$= 45° + 45° \qquad (\because \angle BDA = \angle ADB = 45°)$$
$$= 90°$$

Hence, ∠D makes right angle belongs in semi-circle therefore AC is a diameter of the circle.

(ii) $\angle ACB = \angle ADB$ (Angles in the same segment of a circle)

$\angle ACB = 45°$ $(\because \angle ADB = 45°)$ **Ans.**

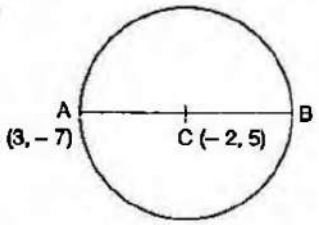
(c) (i) The length of radius AC $= \sqrt{(-2-3)^2 + (5+7)^2}$

$= \sqrt{(-5)^2 + (12)^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13.$ **Ans.**



(ii) Let the point of B be (x, y).

Given C is the mid-point of AB. Therefore

$$-2 = \frac{3+x}{2}$$
$$\Rightarrow \quad 3 + x = -4$$
$$\Rightarrow \quad x = -4 - 3 = -7$$
$$\text{and} \quad 5 = \frac{-7+y}{2}$$
$$\Rightarrow \quad 10 = -7 + y$$
$$y = 17$$

Hence, the co-ordinate of B (–7, 17). **Ans.**

Question 4.

(a) *Solve the following equation and calculate the answer correct to two decimal places :*

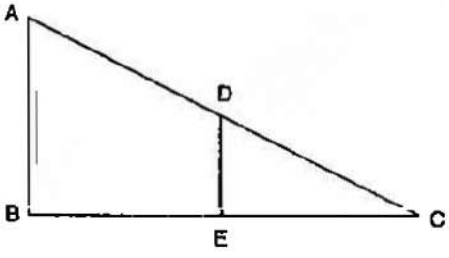
$$x^2 - 5x - 10 = 0.$$ **[3]**

(b) *In the given figure, AB and DE are perpendicular to BC.*

(i) *Prove that Δ ABC ~ Δ DEC*

(ii) *If AB = 6 cm, DE = 4 cm and AC = 15 cm. Calculate CD.*

(iii) *Find the ratio of the area of Δ ABC : area of Δ DEC.* **[3]**



(c) *Using graph paper, plot the points A(6, 4) and B(0, 4).*

(i) *Reflect A and B in the origin to get the images A′ and B′.*

(ii) *Write the co-ordinates of A′ and B′.*

(iii) *State the geometrical name for the figure ABA′B′.*

(iv) *Find its perimeter.* **[4]**

Solution :

(a) *Given* : $x^2 - 5x - 10 = 0$

Here, $a = 1$, $b = -5$ and $c = -10$

$\therefore \quad D = b^2 - 4ac$

$= (-5)^2 - 4 \times 1 \times -10$

$D = 25 + 40 = 65$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{5 \pm \sqrt{65}}{2 \times 1} = \frac{5 \pm 8{\cdot}06}{2}$$

$$= \frac{5 + 8{\cdot}06}{2}, \frac{5 - 8{\cdot}06}{2}$$

$$= \frac{13{\cdot}06}{2}, -\frac{3{\cdot}06}{2}$$

$x = 6{\cdot}53, -1{\cdot}53$ **Ans.**

(b) (i) From Δ ABC and Δ DEC,

$\angle$ ABC = $\angle$ DEC = 90° (Given)

and $\angle$ ACB = $\angle$ DCE = Common

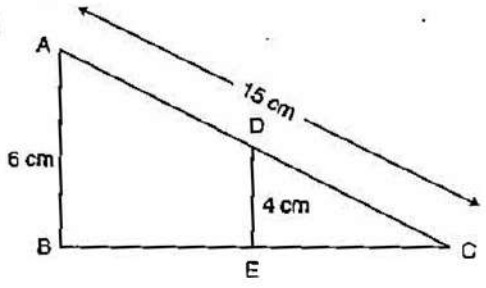
$\therefore \quad \Delta$ ABC ~ Δ DEC (By AA similarity)

(ii) In Δ ABC and Δ DEC,

Δ ABC ~ Δ DEC (proved in (i) part)

$\therefore \quad \frac{AB}{DE} = \frac{AC}{CD}$

Given : AB = 6 cm, DE = 4 cm, AC = 15 cm,



$\therefore \quad \frac{6}{4} = \frac{15}{CD}$

$\Rightarrow \quad 6 \times CD = 15 \times 4$

$\Rightarrow \quad CD = \frac{60}{6}$

$\Rightarrow \quad CD = 10$ cm. **Ans.**

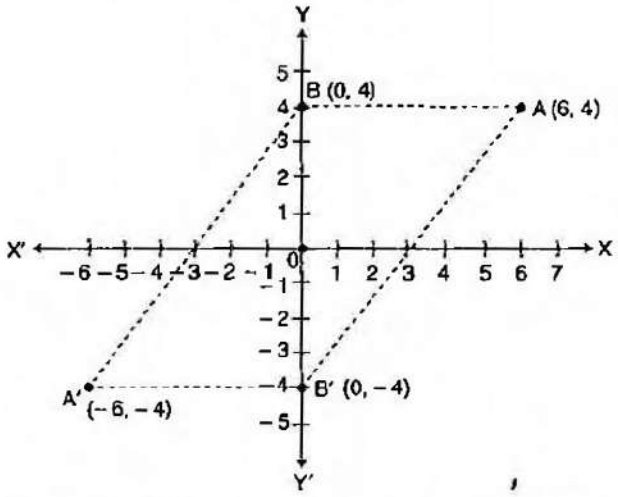
(iii) $\frac{\text{Area of } \Delta \text{ ABC}}{\text{Area of } \Delta \text{ DEC}} = \frac{AB^2}{DE^2}$ ($\because \Delta$ ABC ~ Δ DEC)

$= \frac{(6)^2}{(4)^2}$

$= \frac{3{\cdot}6}{16} = \frac{9}{4}$

$\therefore$ Area of Δ ABC : Area of Δ DEC = 9 : 4. **Ans.**

(c) (i) Please See Graph:



(ii) Reflection of A′ and B′ in the origin = A′ (–6, –4) and B′ (0, –4)

(iii) The geometrical name for the figure AB A′B′ is a parallelogram.

(iv) From the graph, AB = 6 cm, BB′ = 8 cm.

In Δ A BB′

$$(AB')^2 = AB^2 + (BB')^2$$
$$= (6)^2 + (8)^2 = 36 + 64$$
$$= 100$$
$$AB' = 10 = A'B \qquad \text{(AB A′ B′ is a parallelogram)}$$
$$\text{Perimeter of } ABA'B' = A'B' + AB' + AB + A'B$$
$$= 6 + 10 + 6 + 10$$
$$= 32 \text{ units.} \qquad \textbf{Ans.}$$

SECTION B [40 Marks]

*Answer any **four** Questions in this Section.*

Question 5.

(a) *Solve the following inequation, write the solution set and represent it on the number line :*

$$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6},\ x \in R \qquad \textbf{[3]}$$

(b) *Mr. Britto deposits a certain sum of money each month in a Recurring Deposit Account of a bank. If the rate of interest is of 8% per annum and Mr. Britto gets ₹ 8088 from the bank after 3 years, find the value of his monthly instalment.* **[3]**

(c) *Salman buys 50 shares of face value ₹ 100 available at ₹ 132.*

(i) *What is his investment ?*

(ii) *If the dividend is 7.5%, what will be his annual income ?*

(iii) *If he wants to increase his annual income by ₹ 150, how many extra shares should he buy ?* **[4]**

Solution :

(a) *Given* : $-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$

Taking L.C.M. of 3, 2 and 6 is 6.

$$-\frac{x}{3} \times 6 \le \frac{x}{2} \times 6 - \frac{4}{3} \times 6 < \frac{1}{6} \times 6$$

$$-2x \le 3x - 8 < 1$$

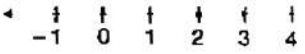
$\Rightarrow \quad -2x \le 3x - 8$ and $3x - 8 < 1$

$\Rightarrow \quad 8 \le 3x + 2x \quad \Rightarrow \quad 3x < 1 + 8$

$\Rightarrow \quad 8 \le 5x \quad 3x < 9$

$\Rightarrow \quad \frac{8}{5} \le x \quad \Rightarrow \quad x < 3$

$\therefore$ The solution set is $\{x : 1{\cdot}6 \le x \le 3, x \in R\}$



Number line

(b) Let the monthly instalment be ₹ x

Given : Maturity amount = ₹ 8,088, Time (n) = 3 years = 3 × 12 months = 36 months, Rate (R) = 8% p.a.

$$\text{Principle for one month} = P \times \frac{n(n+1)}{2}$$

$$= \frac{x \times 36 \times 37}{2}$$

$$= 18 \times 37x$$

$$\text{Interest} = \frac{18 \times 37x \times 8 \times 1}{100 \times 12} \qquad \left[\because I = \frac{PRT}{100}\right]$$

$$= \frac{444x}{100}$$

$$\text{Actual sum deposited} = 36x$$

$$\text{Maturity amount} = \text{Interest} + \text{Actual sum deposited}$$

$$8{,}088 = \frac{444x}{100} + 36x$$

$$8{,}088 = \frac{4{,}044x}{100}$$

$$x = \frac{8{,}088 \times 100}{4{,}044} = 200$$

Hence, the monthly instalment be ₹ 200. **Ans.**

(c) Number of shares = 50

Face value of each share = ₹ 100

Market value of each share = ₹ 132

Total face value = ₹ 100 × 50

= ₹ 5,000

(i) Total investment = ₹ 132 × 50

= ₹ 6,600 **Ans.**

(ii) Rate of dividend = 7·5%

$$\text{Annual income} = ₹\frac{5{,}000 \times 7{\cdot}5}{100}$$

$$= ₹\,375$$ **Ans.**

(iii) Let extra share should he buy be x.

then total number of shares $= 50 + x$

Total face value $= ₹\,100 \times (50 + x)$

$$\therefore \quad \text{Annual income} = ₹\frac{100 \times (50 + x) \times 7{\cdot}5}{100}$$

$$= (50 + x) \times 7{\cdot}5$$

$$\therefore \quad (50 + x) \times 7{\cdot}5 = 375 + 150$$

$$50 + x = \frac{525}{7{\cdot}5} = 70$$

$$x = 70 - 50$$

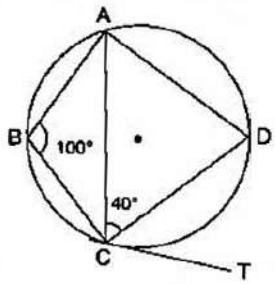
$$x = 20$$

Hence, the extra shares should be buy = 20. **Ans.**

Question 6.

(a) *Show that* $\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A}$ **[3]**

(b) *In the given circle with centre O, $\angle ABC = 100°$, $\angle ACD = 40°$ and CT is a tangent to the circle at C. Find $\angle ADC$ and $\angle DCT$.* **[3]**



(c) *Given below are the entries in a Savings Bank A/c pass book :*

Date	Particulars	Withdrawals	Deposit	Balance
Feb. 8.	*B/F*	—	—	₹ *8,500*
Feb. 18	*To self*	₹ *4,000*	—	—
April 12	*By cash*	—	₹ *2,230*	—
June 15	*To self*	₹ *5,000*	—	—
July 8	*By cash*	—	₹ *6,000*	—

Calculate the interest for six months from February to July at 6% p.a. **[4]**

Solution :

(a) L.H.S. $= \sqrt{\frac{1-\cos A}{1+\cos A}}$

Multiplying by $\sqrt{1+\cos A}$ in numerator and denominator

$$= \sqrt{\frac{1-\cos A}{1+\cos A}} \times \sqrt{\frac{1+\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{(1-\cos A)(1+\cos A)}{(1+\cos A)(1+\cos A)}}$$

$$= \sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}}$$

$$= \sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}$$

$$= \frac{\sin A}{1+\cos A} = \text{R.H.S.}$$ **Proved**

(b) *Given* : $\angle ABC = 100°$

We know that,

$\angle ABC + \angle ADC = 180°$ (The sum of opposite angles in a cyclic quadrilateral = 180°)

$\therefore$ $100° + \angle ADC = 180°$

$\angle ADC = 180° - 100°$

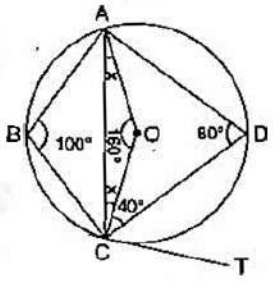
$\angle ADC = 80°$

Join OA and OC, we have a isosceles Δ OAC,

$\because$ $OA = OC$ (Radii of a circle)

$\therefore$ $\angle AOC = 2 \times \angle ADC$ (by theorem)

or $\angle AOC = 2 \times 80° = 160°$



In Δ AOC,

$\angle AOC + \angle OAC + \angle OCA = 180°$

$160° + \angle OCA + \angle OCA = 180°$ [$\because \angle OAC = \angle OCA$]

$2\angle OCA = 20°$

$\angle OCA = 10°$

$\angle OCA + \angle OCD = 40°$

$10° + \angle OCD = 40°$

$\therefore$ $\angle OCD = 30°$

Hence, $\angle OCD + \angle DCT = \angle OCT$

$\because$ $\angle OCT = 90°$

(The tangent at a point to circle is $\perp$ to the radius through the point to contant)

$30° + \angle DCT = 90°$

$\therefore$ $\angle DCT = 60°$ **Ans.**

(c)

Date	Particulars	Withdrawals	Deposit	Balance
Feb. 8	B/F	—	—	₹ 8,500
Feb. 18	To self	₹ 4,000	—	₹ 4,500
April 12	By cash	—	₹ 2,230	₹ 6,730
June 15	To self	₹ 5,000	—	₹ 1,730
July 8	By cash	—	₹ 6,000	₹ 7,730

Principal for the month of Feb. = ₹ 4,500

Principal for the month of March = ₹ 4,500

Principal for the month of April	= ₹	4,500
Principal for the month of May	= ₹	6,730
Principal for the month of June	= ₹	1,730
Principal for the month of July	= ₹	7,730
Total principal from the month of Feb. to July	= ₹	29,690

$$\text{Time} = \frac{1}{12}\text{ years}$$

$$\text{Rate of interest} = 6\%$$

$$\text{Interest} = \frac{P \times R \times T}{100}$$

$$= \frac{29690 \times 6 \times 1}{100 \times 12}$$

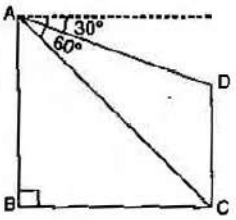
$$= ₹\,148{\cdot}45$$ **Ans.**

Question 7.

(a) *In Δ ABC, A(3, 5), B(7, 8) and C(1, −10). Find the equation of the median through A.* **[3]**

(b) *A shopkeeper sells an article at the listed price of ₹ 1,500 and the rate of VAT is 12% at each stage of sale. If the shopkeeper pays a VAT of ₹ 36 to the Government, what was the price, inclusive of Tax, at which the shopkeeper purchased the article from the wholesaler ?* **[3]**

(c) *In the figure given, from the top of a building AB = 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find :*



(i) *The horizontal distance between AB and CD.*

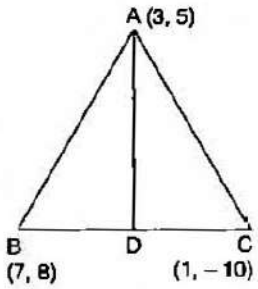
(ii) *The height of the lamp post.* **[4]**

Solution :

(a) Here D is mid point of BC. (Given)

$$\therefore \quad \text{The co-ordinate of D} = \left(\frac{7+1}{2}, \frac{8-10}{2}\right)$$

$$= (4, -1)$$



Now equation of median AD,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here, $x_1 = 3, y_1 = 5, x_2 = 4, y_2 = -1$

$$y - 5 = \frac{-1-5}{4-3}(x-3)$$

$$y - 5 = \frac{-6}{1}(x-3)$$

$$y - 5 = -6x + 18$$

$$y = -6x + 18 + 5$$
$$y = -6x + 23$$
$$6x + y - 23 = 0 \quad \textbf{Ans.}$$

(b) Listed price of an article = ₹ 1,500

Rate of VAT = 12%

$$\text{VAT on the article} = \frac{12}{100} \times 1500$$
$$= ₹\,180$$

Let C.P. of this article be x, then

$$\text{VAT} = \frac{12}{100} \times x$$
$$= ₹\,\frac{12x}{100}$$

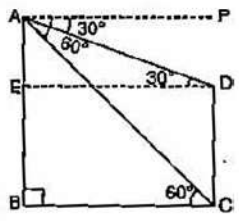
If the shopkeeper pays a VAT = ₹ 36

Then $$180 - \frac{12x}{100} = 36$$
$$\frac{18000 - 12x}{100} = 36$$
$$18000 - 12x = 3600$$
$$\therefore \quad 12x = 18000 - 3600 = 14{,}400$$
$$x = ₹\,1{,}200$$

∴ The price at which the shopkeeper purchased the article inclusive of sales tax

$$= 1{,}200 + \frac{12}{100} \times 1{,}200$$
$$= 1{,}200 + 144$$
$$= ₹\,1{,}344 \quad \textbf{Ans.}$$

(c) *Given* : AB = 60 m



$\because \quad \angle PAC = 60°$

$\therefore \quad \angle PAC = \angle BCA$

(i) Now in ΔABC,

$$\tan 60° = \frac{AB}{BC}$$
$$\sqrt{3} = \frac{60}{BC}$$
$$\Rightarrow \quad \sqrt{3}\,BC = 60$$
$$\Rightarrow \quad BC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$BC = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$$

Hence, the horizontal distance between AB and CD = $20\sqrt{3}$ m. **Ans.**

(ii) Let AE = x and proved above BC = $20\sqrt{3}$ m

$\therefore \quad BC = ED = 20\sqrt{3}$

Now in Δ AED,

$$\tan 30^\circ = \frac{AE}{ED}$$

$$\frac{1}{\sqrt{3}} = \frac{AE}{20\sqrt{3}}$$

$\Rightarrow \quad \sqrt{3}\,AE = 20\sqrt{3}$

$\Rightarrow \quad AE = 20$ m

now $\quad EB = AB - AE$

$\therefore \quad EB = 60 - 20 \Rightarrow 40$ m

$\because \quad EB = CD$

$\therefore \quad CD = 40$ m

Hence, the height of the lamp post = 40 m. **Ans.**

Question 8.

(a) *Find x and y if* $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$ **[3]**

(b) *A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast.* **[3]**

(c) *Without solving the following quadratic equation, find the value of 'p' for which the given equation has real and equal roots :*

$$x^2 + (p-3)x + p = 0$$ **[4]**

Solution :

(a) *Given :*

$$\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 2x + 3x \\ 2y + 4y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 5x \\ 6y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$\therefore \quad 5x = 5 \Rightarrow x = 1$

and $\quad 6y = 12 \Rightarrow y = 2$

Hence, $x = 1$ and $y = 2$ **Ans.**

(b) Radius of a solid sphere, $r = 15$ cm

Volume of a solid sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi\,(15)^3$ cm^3.

Now, radius of right circular cone $= 2{\cdot}5$ cm

and height, $h = 8$ cm.

Volume of right circular cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\,(2{\cdot}5)^2 \times 8$

$$\therefore \quad \text{The number of cones} = \frac{\text{Volume of a sphere}}{\text{Volume of a cone}}$$

$$= \frac{\frac{4}{3}\pi \times (15)^3}{\frac{1}{3}\pi (2{\cdot}5)^2 \times 8}$$

$$= \frac{15 \times 15 \times 15}{2{\cdot}5 \times 2{\cdot}5 \times 2}$$

$$= 270 \qquad \textbf{Ans.}$$

(c) Given equation $x^2 + (p-3)x + p = 0$

$\because$ Roots are real and equal, then

$$b^2 - 4ac = 0$$

Here we compare the coefficients of a, b and c with the equation $ax^2 + bx + c = 0$.

$$a = 1, b = p - 3 \text{ and } c = p$$

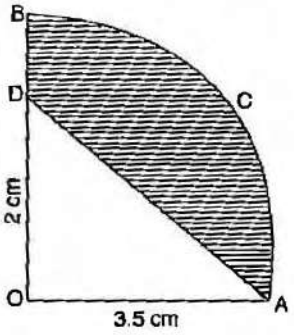
Now putting the values of a, b and c in equation

$$(p-3)^2 - 4 \times 1 \times p = 0$$
$$p^2 + 9 - 6p - 4p = 0$$
$$p^2 + 9 - 10p = 0$$
$$p^2 - 10p + 9 = 0$$
$$p^2 - 9p - p + 9 = 0$$
$$p(p-9) - 1(p-9) = 0$$
$$\Rightarrow \quad (p-9)(p-1) = 0$$

Hence, $p = 9$ or 1 **Ans.**

Question 9.

(a) *In the figure alongside, OAB is a quadrant of a circle. The radius OA = 3.5 cm and OD = 2 cm. Calculate the area of the shaded portion.* $\left(\text{Take } \pi = \frac{22}{7}\right)$ **[3]**



(b) *A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball, find the number of black balls in the box.* **[3]**

(c) *Find the mean of the following distribution by step deviation method :* **[4]**

Class Interval	*20–30*	*30–40*	*40–50*	*50–60*	*60–70*	*70–80*
Frequency	*10*	*6*	*8*	*12*	*5*	*9*

Solution :

(a) Radius of quadrant OACB, $r = 3{\cdot}5$ cm

$$\text{Area of quadrant OACB} = \frac{1}{4}\pi r^2$$
$$= \frac{1}{4} \times \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5$$
$$= 9{\cdot}625 \text{ cm}^2.$$

Here, $\angle AOD = 90°$

Then $\text{area of } \Delta AOD = \frac{1}{2} \times \text{base} \times \text{height}$

Base = 3·5 cm and height = 2 cm

$$= \frac{1}{2} \times 3{\cdot}5 \times 2 = 3{\cdot}5 \text{ cm}^2.$$

$$\text{Area of shaded portion} = \text{Area of quadrant} - \text{Area of triangle}$$
$$= 9{\cdot}625 - 3{\cdot}5$$
$$= 6{\cdot}125 \text{ cm}^2. \quad \textbf{Ans.}$$

(b) Let the number of black balls be x, then

$$\text{Total number of balls} = 30 + x$$

Thus, the prabability of black balls $= \frac{x}{30+x}$.

and the probability of white balls $= \frac{30}{30+x}$

Given : $\text{Probability of black ball} = \frac{2}{5} \times \text{probability of white ball}$

$$\frac{x}{30+x} = \frac{2}{5} \times \frac{30}{x+30}$$
$$5x = 60$$
$$x = 12 \quad \textbf{Ans.}$$

Hence, the number of black balls = 12.

(c)

C.I.	Frequency (f_i)	Mid-value (x)	$d_i = \frac{x-a}{h}$	$f_i d_i$
20–30	10	25	–2	–20
30–40	6	35	–1	–6
40–50	8	45	0	0
50–60	12	55	1	12
60–70	5	65	2	10
70–80	9	75	3	27
	$\Sigma f_i = 50$			$\Sigma f_i d_i = 23$

Here, $a = 45$ and $h = 10$

$$\text{Mean} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} \times h$$
$$= 45 + \frac{23}{50} \times 10$$
$$= 45 + 4{\cdot}6 = 49{\cdot}6. \quad \textbf{Ans.}$$

Question 10.

(a) *Using a ruler and compasses only :*

(i) *Construct a triangle ABC with the following data :*
AB = 3.5 cm, BC = 6 cm and ∠ABC = 120°

(ii) *In the same diagram, draw a circle with BC as diameter. Find a point P on the circumference of the circle which is equidistant from AB and BC.*

(iii) *Measure ∠BCP.* **[3]**

(b) *The mark obtained by 120 students in a test are given below :*

Marks	No. of Students
0–10	5
10–20	9
20–30	16
30–40	22
40–50	26
50–60	18
60–70	11
70–80	6
80–90	4
90–100	3

Draw an ogive for the given distribution on a graph sheet.
Using suitable scale for ogive to estimate the following :

(i) The median.

(ii) The number of students who obtained more than 75% marks in the test.

(iii) The number of students who did not pass the test if minimum marks required to pass is 40. **[6]**

Solution :

(a) Steps of Construction :

(i) Draw a line BC = 6 cm.

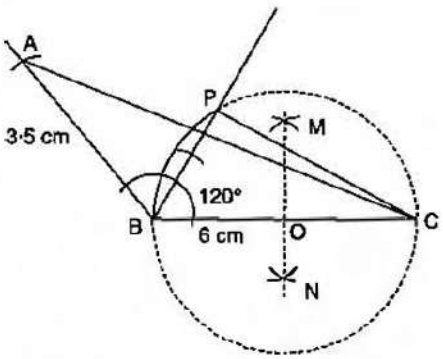
(ii) With the help of the point B, draw ∠ ABC = 120°

(iii) Taking radius 3·5 cm cut BA = 3·5 cm.

(iv) Join A to C.

(v) Draw ⊥ bisector MN of BC.

(vi) Draw a circle O as centre and OC as radius.



(vii) Draw angle bisector of ∠ ABC which intersects circle at P.

(viii) Join BP and CP.

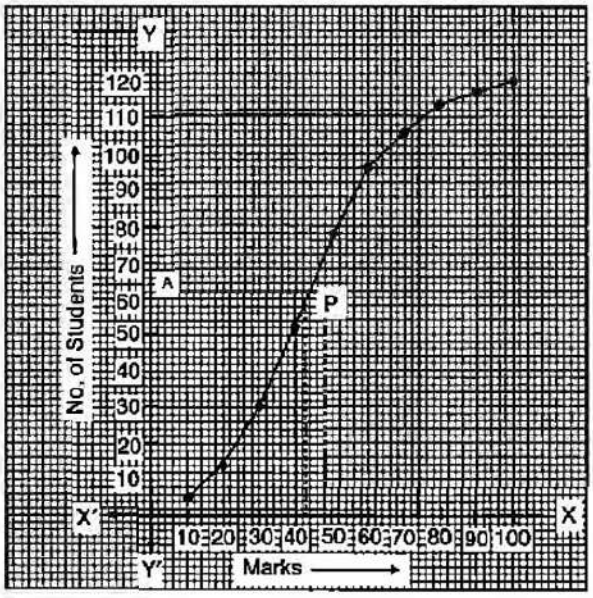
(ix) Now, ∠ BCP = 30°.

(b)

Marks	No. of Students (f)	Cumulative Frequency
0–10	5	5
10–20	9	14
20–30	16	30
30–40	22	52
40–50	26	78
50–60	18	96
60–70	11	107
70–80	6	113
80–90	4	117
90–100	3	120
	$n = 120$	

On the graph paper, we plot the following points :

(10, 5), (20, 14), (30, 30), (40, 52), (50, 78), (60, 96), (70, 107), (80, 113), (90, 117), (100, 120).



(i) $$\text{Median} = \left(\frac{n}{2}\right)^{\text{th}} \text{ term} \qquad [\because n = 120, \text{ even}]$$

$$= \frac{120}{2} = 60^{\text{th}} \text{ term}$$

From the graph 60th term = 42 **Ans.**

(ii) The number of students who obtained more than 75% marks in test

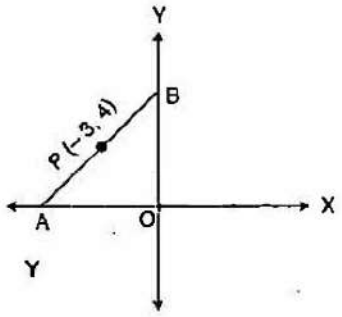
$= 120 - 110$

$= 10.$ **Ans.**

(iii) The number of students who did not pass the test if the minimum pass marks 40 = 52. **Ans.**

Question 11.

(a) *In the figure given below, the line segment AB meets X-axis at A and Y-axis at B. The point P(–3, 4) on AB divides it in the ratio 2 : 3. Find the coordinates of A and B.*



(b) *Using the properties of proportion, solve for x, given*

$$\frac{x^4+1}{2x^2} = \frac{17}{8}$$ **[3]**

(c) *A shopkeeper purchases a certain number of books for ₹ 960. If the cost per book was ₹ 8 less, the number of books that could be purchased for ₹ 960 would be 4 more. Write an equation, taking the original cost of each book to be ₹ x, and solve it to find the original cost of the books.* **[4]**

Solution :

(a) Let the co-ordinates of A and B be $(x, 0)$ and $(0, y)$

$\because$ The co-ordinates of a point P (–3, 4) on AB divides it in the ratio 2 : 3.

i.e., $AP : PB = 2 : 3$

By using section formula, we get

$$-3 = \frac{2\times 0 + 3\times x}{2+3} \qquad \left[\because x = \frac{m_1x_2 + m_2x_1}{m_1+m_2}\right]$$

$$-3 = \frac{3x}{5} \Rightarrow 3x = -15$$

$$\Rightarrow \quad x = -5$$

and $$4 = \frac{2\times y + 3\times 0}{2+3} \qquad \left[\because y = \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right]$$

$$4 = \frac{2y}{5} \Rightarrow 2y = 20$$

$$\Rightarrow \quad y = 10$$

Hence, the co-ordinates of A and B are (–5, 0) and (0, 10). **Ans.**

(b) *Given :* $\frac{x^4+1}{2x^2} = \frac{17}{8}$

By using componendo and dividendo, we get

$$\frac{x^4+1+2x^2}{x^4+1-2x^2} = \frac{17+8}{17-8}$$

$$\left(\frac{x^2+1}{x^2-1}\right)^2 = \frac{25}{9}$$

$$\left(\frac{x^2+1}{x^2-1}\right)^2 = \left(\frac{5}{3}\right)^2$$

Taking square root on both sides, we get

$$\frac{x^2+1}{x^2-1} = \frac{5}{3}$$

$\Rightarrow \quad 5x^2-5 = 3x^2+3$

$\Rightarrow \quad 5x^2-3x^2 = 3+5$

$\Rightarrow \quad 2x^2 = 8 \quad \Rightarrow \quad x^2 = 4$

$\Rightarrow \quad x = \pm 2$ **Ans.**

(c) Given the original cost of each book be ₹ x.

Total cost = ₹ 960 (Given)

Number of books for 960 = $\frac{960}{x}$

If the cost per book was ₹ 8 less, (*i.e.*, $x-8$) then

Number of books = $\frac{960}{x-8}$

According to question,

$$\frac{960}{x-8} = \frac{960}{x} + 4$$

$$\frac{960}{x-8} - \frac{960}{x} = 4$$

$$960\left[\frac{x-x+8}{x(x-8)}\right] = 4$$

$$\frac{8}{x^2-8x} = \frac{1}{240}$$

$\Rightarrow \quad x^2-8x = 1{,}920$

$x^2-8x-1{,}920 = 0$ **Ans.**

$\Rightarrow \quad x^2-48x+40x-1{,}920 = 0$

$\Rightarrow \quad x(x-48)+40(x-48) = 0$

$\Rightarrow \quad (x-48)(x+40) = 0$

$x-48 = 0$ or $x+40 = 0$

$x = 48$ or $x = -40$

$\because -40$ is not possible.

Hence, the original cost of each book = ₹ 48. **Ans**

ICSE QUESTION PAPER (2014)

MATHEMATICS

(Two hours and a half)

Answers to this Paper must be written on the paper provided separately.

*You will **not** be allowed to write during the first **15** minutes.*

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*

All working, including rough work, must be clearly shown and must be done on the sam sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables are provided.

SECTION A (40 Marks)

*Attempt **all** questions from this Section.*

Question 1

(a) Ranbir borrows ₹ 20,000 at 12% per annum compound interest. If he repays ₹ 8400 at the end of the first year and ₹ 9680 at the end of the second year, find the amount of loan outstanding at the beginning of the third year. [

(b) Find the values of x, which satisfy the inequation $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2$, $x \in W$. Graph the solution set on the number line.

(c) A die has 6 faces marked by the given numbers as shown below:

1	2	3	–1	–2	–3

The die is thrown once. What is the probability of getting

(i) a positive integer.

(ii) an integer greater than –3.

(iii) the smallest integer.

Question 2

(a) Find x, y if $\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}$.

(b) Shahrukh opened a Recurring Deposit Account in a bank and deposited ₹800 per month for 1 ½ years. If he received ₹15,084 at the time of maturity, find the rate of interest per annum.

(c) Calculate the ratio in which the line joining A(-4, 2) and B(3, 6) is divided by point P(x, 3). Also find (i) x (ii) Length of AP.

Question 3

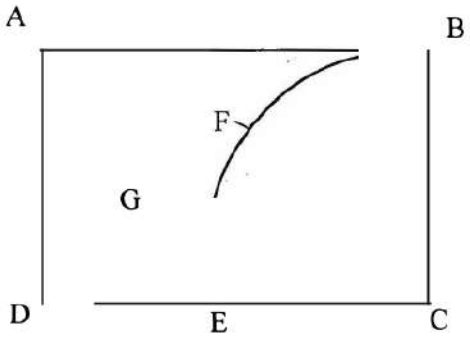
(a) Without using trigonometric tables, evaluate

$\sin^2 34° + \sin^2 56° + 2 \tan 18° \tan 72° - \cot^2 30°$

(b) Using the Remainder and Factor Theorem, factorise the following polynomial:

$x^3 + 10x^2 - 37x + 26$.

(c) In the figure given below, ABCD is a rectangle. AB = 14cm, BC = 7cm. From the rectangle, a quarter circle BFEC and a semicircle DGE are removed. Calculate the area of the remaining piece of the rectangle.(Take $\pi = 22/7$)



Question 4

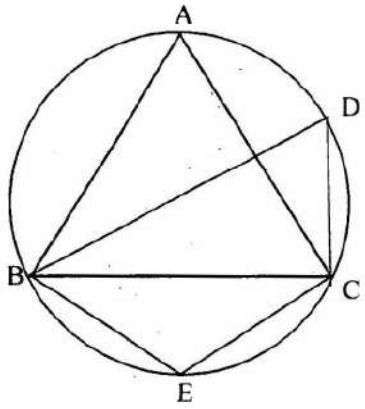
(a) The numbers 6, 8, 10, 12, 13, and x are arranged in an ascending order. If the mean of the observations is equal to the median, find the value of x.

(b) In the figure, $\angle DBC = 58°$. BD is a diameter of the circle. Calculate:

(i) $\angle BDC$

(ii) $\angle BEC$

(iii) $\angle BAC$



(c) Use graph paper to answer the following questions. (Take 2cm = 1 unit on both axis)

(i) Plot the points A(- 4, 2) and B(2, 4)

(ii) A′ is the image of A when reflected in the y-axis. Plot it on the graph paper and write the coordinates of A′.

(iii) B′ is the image of B when reflected in the line AA′. Write the coordinates of B′ .

(iv) Write the geometric name of the figure ABA′B′.

(v) Name a line of symmetry of the figure formed.

SECTION B (40 Marks)

*Attempt any **four** questions from this Section*

Question 5

(a) A shopkeeper bought a washing machine at a discount of 20% from a wholesaler, the printed price of the washing machine being ₹18,000. The shopkeeper sells it to a consumer at a discount of 10% on the printed price. If the rate of sales tax is 8%, find:

(i) the VAT paid by the shopkeeper.

(ii) the total amount that the consumer pays for the washing machine.

(b) If $\frac{x^2+y^2}{x^2-y^2} = \frac{17}{8}$, then find the value of:

(i) $x : y$

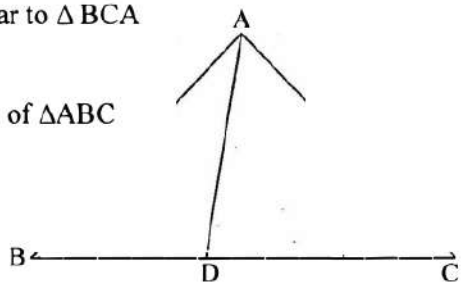
(ii) $\frac{x^3+y^3}{x^3-y^3}$

(c) In ΔABC, $\angle ABC = \angle DAC$. AB = 8cm, AC = 4cm, AD = 5cm.

(i) Prove that ΔACD is similar to ΔBCA

(ii) Find BC and CD

(iii) Find area of ΔACD : area of ΔABC



Question 6

(a) Find the value of 'a' for which the following points A(a, 3), B (2, 1) and C(5, a) are collinear. Hence find the equation of the line.

(b) Salman invests a sum of money in ₹ 50 shares, paying 15% dividend quoted at 20% premium. If his annual dividend is ₹ 600, calculate:

(i) the number of shares he bought.

(ii) his total investment.

(iii) the rate of return on his investment.

(c) The surface area of a solid metallic sphere is 2464 cm^2. It is melted and recast into solid right circular cones of radius 3.5cm and height 7cm. Calculate:

(i) the radius of the sphere.

(ii) the number of cones recast. (Take $\pi = 22/7$)

Question 7

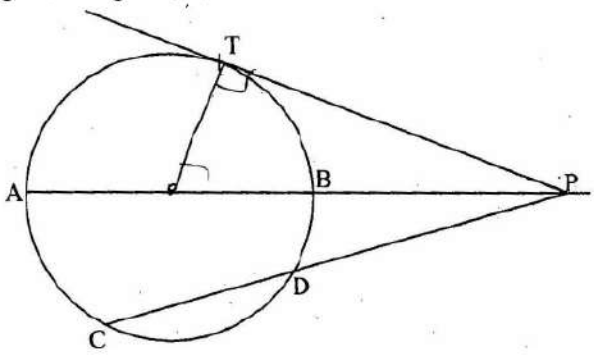
(a) Calculate the mean of the distribution given below using the short cut method.

Marks	11-20	21-30	31-40	41-50	51-60	61-70	71-80
No. of students	2	6	10	12	9	7	4

(b) In the figure given below, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. CD = 7.8cm, PD = 5cm, PB = 4cm. Find:

(i) AB.

(ii) the length of tangent PT.



[3]

(c) Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$.

Find $A^2 + AC - 5B$.

[4]

Question 8

(a) The compound interest, calculated yearly, on a certain sum of money for the second year is ₹1320 and for the third year is ₹1452. Calculate the rate of interest and the original sum of money. [3

(b) Construct a Δ ABC with BC= 6.5 cm, AB= 5.5 cm, AC=5 cm. Construct the incircle of the triangle. Measure and record the radius of the incircle.

(c) (Use a graph paper for this question.) The daily pocket expenses of 200 students in a school are given below:

Pocket expenses (in ₹)	Number of students (frequency)
0 – 5	10
5 – 10	14
10 – 15	28
15 – 20	42
20 – 25	50
25 – 30	30
30 – 35	14
35 – 40	12

Draw a histogram representing the above distribution and estimate the mod from the graph.

Question 9

(a) If $(x - 9) : (3x + 6)$ is the duplicate ratio of 4 : 9, find the value of x.

(b) Solve for x using the quadratic formula. Write your answer correct to tw significant figures. $(x - 1)^2 - 3x + 4 = 0$.

(c) A page from the savings bank account of Priyanka is given below:

Date	Particulars	Amount withdrawn (₹)	Amount deposited (₹)	Balance (₹)
03/04/2006	B/F			4000.00
05/04/2006	By cash		2000.00	6000.00
18/04/2006	By cheque		6000.00	12000.00
25/05/2006	To cheque	5000.00		7000.00
30/05/2006	By cash		3000.00	10000.00
20/07/2006	By self	4000.00		6000.00
10/09/2006	By cash		2000.00	8000.00
19/09/2006	To cheque	1000.00		7000.00

If the interest earned by Priyanka for the period ending September, 2006 i ₹ 175, find the rate of interest.

Question 10

(a) A two digit positive number is such that the product of its digits is 6. If 9 i added to the number, the digits interchange their places. Find the number.

(b) The marks obtained by 100 students in a Mathematics test are given below:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	
No of students	3	7	12	17	23	14	9	6	5	

Draw an ogive for the given distribution on a graph sheet.

Use a scale of 2cm = 10 units on both axis).

Use the ogive to estimate the:

(i) median.

(ii) lower quartile.

(iii) number of students who obtained more than 85% marks in the test.

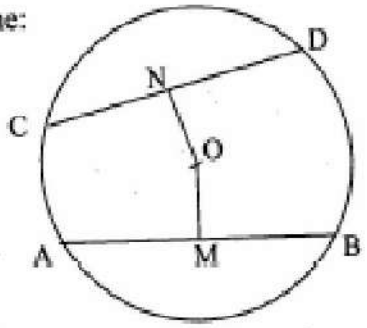
(iv) number of students who did not pass in the test if the pass percentage was 35. [6]

Question 11

(a) In the figure given below, O is the centre of the circle. AB and CD are two chords of the circle. OM is perpendicular to AB and ON is perpendicular to CD. AB = 24cm, OM = 5cm, ON = 12cm. Find the:

(i) radius of the circle.

(ii) length of chord CD. [3]



(b) Prove the identity

$(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$. [3]

(c) An aeroplane at an altitude of 250 m observes the angle of depression of two boats on the opposite banks of a river to be 45° and 60° respectively. Find the width of the river. Write the answer correct to the nearest whole number. [4

ICSE Class 10 Maths Question Paper Solution 2015

Question 1

(a) A shopkeeper bought an article for ₹3,450. He marks the price of the article 16% above the cost price. The rate of sales tax charged on the article is 10%. Find the:

(i) marked price of the article.

(ii) price paid by a customer who buys the article. [3]

(b) Solve the following inequation and write the solution set:

$13x - 5 < 15x + 4 < 7x + 12,\ x \in R$

Represent the solution on a real number line. [3]

(c) Without using trigonometric tables evaluate:

$$\frac{\sin 65°}{\cos 25°} + \frac{\cos 32°}{\sin 58°} - \sin 28°.\ \sec 62° + \text{cosec}^2 30°$$ [4]

Examiners' Comments

(a) Some candidates calculated 10% sales tax on the cost price instead of marked price that led to a number of calculation errors. They also expressed the final customer's price as ₹4402.2 instead of ₹4402.20.

(b) Common errors were made in transposing x terms on one side and constants on the other. Some candidates tried to work out both inequalities simultaneously and hence made errors, both with signs and transposition of terms. Solution to the inequality was not written in the set form as stated in the question. It is necessary to put arrows on both sides of the number line with at least one extra element on each side so as to indicate the continuity of the infinite real number line.

(c) Candidates adopted incorrect methods of conversion of trigonometric ratio of complementary angles, e.g.

Incorrect Method	Correct Method
$\sin 65° = \cos(90° - 25°)$	$\sin 65° = \cos(90° - 65°)$
$\sin 65° = \sin(90° - 65°)$	$\sin 65° = \sin(90° - 25°)$

Candidates were unable to write the value of cosec 30° as 2 and missed out on a number of essential steps that led to incorrect answer.

Suggestions for teachers

Students must be advised to read the question carefully for correct identification of data given and the result that is to be to be found. Further all decimal answers related to money must be written to two decimal places e.g., ₹4402.20

It is advisable to solve the inequation by taking the two inequalities separately, e.g., 13x-5<15x+4<7x+12 to be taken as 13x-5<15x+4 and 7x+12.

Students need to understand the concept that -3x<9 is 3x>-9 and not 3x<-9.

Students must always write the solution in the appropriate set form.

It is necessary to have rigorous practice of complementary angles of trigonometric ratios and the common errors must be highlighted. Emphasis must be given on the concepts of values of special trigonometric angles, e.g., 0°,30°,45°,60° and 90°. Examine assignments to ensure essential steps are being followed, e.g. sin65° must not be directly written as cos25°

MARKING SCHEME

Question 1

(a) C.P. of the article is Rs.3,450/-

(i) M.P. of the article = $3450 + \frac{16}{100} \times 3450 + 552 =$ ` 4002

(ii) S.T. = $\frac{10}{100} \times 4002 =$ `400.20

∴ Price paid by the customer = 4002 + 400.20 = ` 4402.20

(b) $13x - 5 < 15x + 4 < 7x + 12, \quad x \in R$

$13x - 5 < 15x + 4$ | $15x + 4 < 7x + 12$

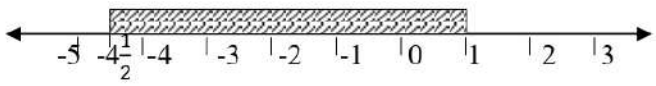
$13x - 15x < 4 + 5$ | $15x - 7x < 12 - 4$

$-2x < 9$ | $8x < 8$

$\therefore x > -4\frac{1}{2}$ | $\therefore x < 1$

Any one correctly transposed

Solution $\left\{x: -4\frac{1}{2} < x < 1, x \in R\right\}$



(c) $\frac{\sin 65°}{\cos 25°} + \frac{\cos 32°}{\sin 58°} - \sin 28° \sec 62° + \text{cosec}^2 30$

$\frac{\cos(90 - 65)}{\cos 25} + \frac{\sin(90 - 32)}{\sin 58} - \cos(90 - 28) \times \sec 62 + (2)^2$

Any one complementary angle correct

$= \frac{\cos 25}{\cos 25} + \frac{\sin 58}{\sin 58} - \cos 62 \times \frac{1}{\cos 62} + 4$

$1 + 1 - 1 + 4 = 5$

Question 2

(a) If $A=\begin{bmatrix}3 & x\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}9 & 16\\ 0 & -y\end{bmatrix}$, find x and y when $A^2 = B$. [3]

(b) The present population of a town is 2,00,000. Its population increases by 10% in the first year and 15% in the second year. Find the population of the town at the end of the two years. [3]

(c) Three vertices of a parallelogram ABCD taken in order are A (3, 6), B (5, 10) and C (3, 2) find:

(i) the coordinates of the fourth vertex D.

(ii) length of diagonal BD.

(iii) equation of side AB of the parallelogram ABCD. [4]

Examiners' Comments

(a) Some candidates took entries of matrix A^2 by squaring the elements of A instead of finding the product $A \times A$. Thus got $A^2 = \begin{bmatrix}9 & x^2\\ 0 & 1\end{bmatrix}$ instead of $\begin{bmatrix}9 & 4x\\ 0 & 1\end{bmatrix}$.Some added the elements and then equated the corresponding elements of $\begin{bmatrix}9 & 4x\\ 0 & 1\end{bmatrix}$ and $\begin{bmatrix}9 & 16\\ 0 & -y\end{bmatrix}$hence got the incorrect values of x and y. e.g. $9 + 4x = 9 + 16$

(b) Some candidates lacked basic clarity about the concepts of compound interest. Due to incorrect concept they took the principal of Second year as ₹2,00,000 instead of ₹2,20,000 which is the amount at the end of first year.

(c) Candidates adopted steps that led to cumbersome calculations The numerical problem could easily be solved by using midpoint theorem or by finding slopes, e.g., $\frac{5+x}{2} = \frac{3+3}{2}$ $(equating\ the\ x-coordinates\ of midpoint\ of\ AC\ and\ BD\ )$ or by equating slopes of AB & DC, e.g. $\frac{2-y}{3-x} = \frac{6-10}{-5+3}$.

If worked out by equating two distances AD and BC then working becomes tedious and answer to the sum is incorrect, e.g., $\sqrt{(3-x)^2+(y-6)^2} = \sqrt{(5-3)^2+(10-2)^2}$

Suggestions for teachers

- Multiplication of matrices must be made clear and to understand that A^2 is the matrix product A×A and is not a matrix whose elements are the square of the elements of A.
- Basic concepts of Compound Interest calculation needs to be made clear. Students must be able to differentiate between Simple Interest and Compound Interest.
- Concepts on Coordinate Geometry need to be explained in different methods of solving a particular numerical problem. This would assist students avoid the cumbersome approaches of solving a numerical problem.

Marking Scheme

Question 2

(a) $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$

$A^2 = B$

$\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$

$\begin{bmatrix} 9+0 & 3x+x \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$

$\therefore \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$

$\therefore 4x = 16 \Rightarrow x = 4$

$1 = -y \Rightarrow y = -1$

Equating both correctly

(b) Present population 2,00,000, rate of increase 10% and 15%

Method 1:

Population after 2 years

$= 200000\left(1 + \frac{10}{100}\right)\left(1 + \frac{15}{100}\right)$

$= 200000 \times \frac{11}{10} \times \frac{23}{20} = 253000$

Simplifying

Method 2:

Population at the end of 1st year

$= 200000 + \frac{10}{100} \times 200000$

$= 220000$

Population at the end of 2nd year

$= 220000 + \frac{15}{100} \times 220000$

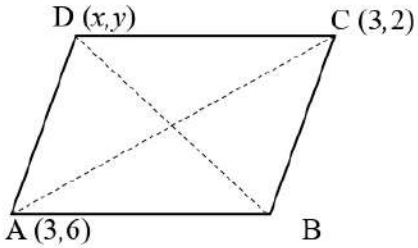
$= 220000 + 33000 = 253000$

(c) (i) Mid point of AC = Mid point of BD

$\therefore \frac{3+3}{2} = \frac{5+x}{2}$ and $\frac{6+2}{2} = \frac{10+y}{2}$

$\therefore x = 1 \qquad y = -2$

$\therefore$ D(1, –2)



(ii) Length of BD :

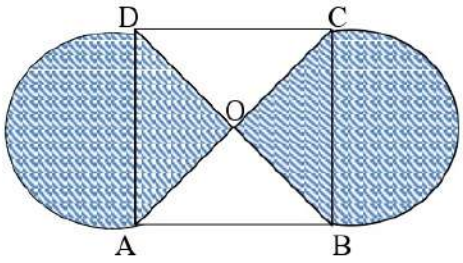
$\sqrt{(10+2)^2 + (5-1)^2} = \sqrt{12^2 + 4^2} = \sqrt{160} = 4\sqrt{10}$ units

(iii) Slope of AB = $\frac{10-6}{5-3} = \frac{4}{2} = 2$

$\therefore$ Eqn of AB : $y - 6 = 2(x - 3)$ *or* $y - 6 = 2x - 6$

Question 3

(a) In the given figure, ABCD is a square of side 21 cm. AC and BD are two diagonals of the square. Two semi circles are drawn with AD and BC as diameters. Find the area of the shaded region. (Take $\pi = \frac{22}{7}$) [3]

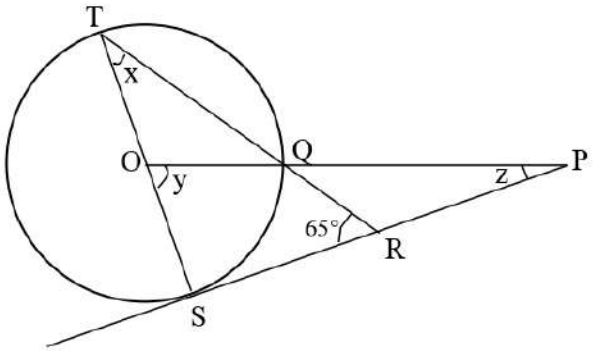


(b) The marks obtained by 30 students in a class assessment of 5 marks is given below:

Marks	0	1	2	3	4	5
No. of Students	1	3	6	10	5	5

Calculate the mean, median and mode of the above distribution. [3]

(c) In the figure given below, O is the centre of the circle and SP is a tangent. If ∠SRT = 65°, find the value of x, y and z.



[4]

Examiners' Comments

(a) Candidates committed calculation errors made as they did not follow the appropriate approach, e.g. (i) taking $\pi = 3.14$ instead of $\frac{22}{7}$ as instructed in the question. This led to lengthy working; (ii) Finding the length of diagonal to get the area of the triangles where as they could have just found it by using the fact that the area of the two triangles $= \frac{1}{2}$ *Area of the Square*. Some made mistakes by taking the radius as 21 instead of 21/2.

(b) Some candidates solved the numerical problem correctly however they incorrectly mentioned mean for median and vice versa. Candidates made errors in calculating fx or finding cumulative frequency c and some calculated fc instead of fx, A few candidates went on to find median graphically for the given non-grouped distribution and arrived at incorrect answers.

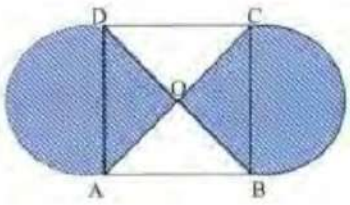
(c) Most candidates failed to cite reasons while solving numerical problems. Some candidates were not versed with the circle properties, hence could not identify the fact $\angle y = 2\angle x$ or $\angle PST = 90°$

Suggestions for teachers

- Students need to be trained on the correct approach of solving Mensuration numerical problems with the value of π taken as 22/7 if given.
- If students had identified that, the area of the two triangles is equal to half the area of the square they need not find the length of the diagonal.
- Insist on instructions to be followed as given in a question. It is essential to draw graphs only when it is specified in the question. Students must be made to understand the meaning of $\sum f$ some students took $\sum f$ as 6 instead of 30.
- Emphasise on the following while teaching: (i) identifying angles correctly. ∠TRS cannot be written as ∠R. (ii) Essential working must be shown, (iii) reasons must be clearly stated.

MARKING SCHEME

Question 3



(a) ABCD is a square of side 21 cm.

$\therefore$ Area of square $= 21^2 = 441$

$\therefore$ Area of the shaded part = area of the two triangular part + Area of the two semi circles.

$\therefore$ Area of shaded part $= \frac{1}{2} \times 441 + \pi r^2 = \frac{1}{2} \times 441 + \frac{22}{7} \times \left(\frac{21}{2}\right)^2$

$= \frac{441}{2} + \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{441}{2} + \frac{693}{2} = \frac{1134}{2} = 567$ sq units

(b)

Marks (x)	Number of students (f)	fx	cf
0	1	0	1
1	3	3	4
2	6	12	10
3	10	30	20
4	5	20	25
5	5	25	30
	30	90	

$\therefore$ Mean $= \frac{\varepsilon fx}{\varepsilon f} = \frac{90}{30} = 3$

Median = 3

Mode = 3

(c) In $\Delta RST, \angle s = 90°$ ($\therefore$ ST is a diameter)

$\therefore x = 180° - (90° + 65°)$; (angles of a triar upto 180°)

$= 25°$

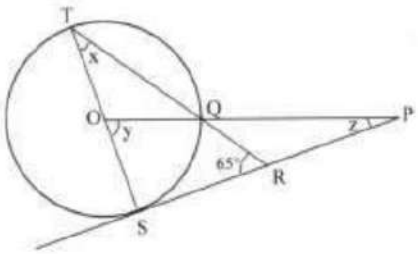
$\angle y = 2\angle x = 2 \times 25° = 50°$

(angle of the centre is double the angl

In the remaining circumference

$x = 180° - (90° + 50°) = 40°$ (With at least one reason)

(angles of a triangle otherwise adds upto 180°)



Question 4

(a) Katrina opened a recurring deposit account with a Nationalised Bank for a period of 2 years. If the bank pays interest at the rate of 6% per annum and the monthly instalment is `1,000, find the:

(i) interest earned in 2 years.

(ii) matured value. [3]

(b) Find the value of 'K' for which x = 3 is a solution of the quadratic equation,

$(K + 2)x^2 - Kx + 6 = 0$.

Thus find the other root of the equation. [3]

(c) Construct a regular hexagon of side 5 cm. Construct a circle circumscribing the hexagon. All traces of construction must be clearly shown. [4]

Examiners' Comments

(a) Errors of candidates were mostly based on formula , application of the formula or calculation error, e.g., (i) *n* is taken as '2' instead of 24 months; (ii) *r* is taken as 6 instead of $\frac{6}{12}$ as interest is being calculated per month ; (iii) First part of question is to find the interest which is equal to ₹1500,
Some candidates found the final answer as matured value =₹25,500 which is the second part of the question hence led to incorrect answers.

(b) Candidates were able to find $k = -4$. Some used the direct method of replacing *x by* 3 and did it easily. On the other hand there were candidates who used the formula method of solving *x* and equating to 3 to find *k*. This led to more working and thereby incorrect answers.
A few candidates were unable to find the other factor which was simply to solve the quadratic equation.

(c) It is necessary to use a ruler and compass and show all traces while doing any geometrical construction. According to the question it is necessary to construct the hexagon and locate the circumcentre by construction and finally to draw the circle. Drawing a circle of 5cm radius and then constructing the hexagon by cutting arcs is incorrect according to the given conditions.

Suggestions for teachers

- Advise students to read the question carefully so as to identify correctly what is the given data and what is required to be found.
- A thorough clarity on concepts to ensure students are confident of attempting the question. They should be able to identify a quadratic equation and hence solve to find the roots, e.g., after substituting k in the equation it becomes the quadratic x^2-2x-3=0, hence (x-3)(x+1)=0 . ∴ other root is -1.
- The question must be read carefully before an attempt is made to answer.

MARKING SCHEME

Question 4

(a) Instalment = `1000, number of months = 12 x 2 = 24, Rate = 6%

(i) $\therefore$ Interest $= \frac{PRT}{100} = \frac{1000 \times 6 \times 24(24+1)}{100 \times 12 \times 2} =$ `1500

(ii) Matured value = Total amount deposited + Interest

=1000 x 24 + 1500 = `25,500

(b) $(K+2)x^2 - Kx + 6 = 0$, $x = 3$ is a root of the equation

$\therefore (K+2)3^2 - K \times 3 + 6 = 0$

$9K + 18 - 3K + 6 = 0$

$\therefore 6K = -24 \; or \; K = -4$

$\therefore$ The equation is $-2x^2 + 4x + 6 = 0$

$x^2 - 2x - 3 = 0$

$\therefore (x-3)(x+1) = 0$

$\therefore x = 3, \; x = -1$

Hence the second root is -1

(c) One side and one 120°

Hexagon

Bisection of one side

Two sides bisection to locate centre and circumcircle

Question 5

(a) Use a graph paper for this question taking 1 cm = 1 unit along both the x and y axis:

(i) Plot the points A(0, 5), B(2, 5), C(5, 2), D(5, -2), E(2, -5) and F(0, -5).

(ii) Reflect the points B, C, D and E on the y-axis and name them respectively as B′, C′, D′ and E′.

(iii) Write the coordinates of B′, C′, D′ and E′.

(iv) Name the figure formed by B C D E E′ D′C′B′.

(v) Name a line of symmetry for the figure formed. [5]

(b) Virat opened a Savings Bank account in a bank on 16th April 2010. His pass book shows the following entries:

Date	Particulars	Withdrawal (`)	Deposit (`)	Balance (`)
April 16, 2010	By cash	-	2500	2500
April 28th	By cheque	-	3000	5500
May 9th	To cheque	850	-	4650
May 15th	By cash	-	1600	6250
May 24th	To cash	1000	-	5250
June 4th	To cash	500	-	4750
June 30th	By cheque	-	2400	7150
July 3rd	By cash	-	1800	8950

Calculate the interest Virat earned at the end of 31st July, 2010 at 4% per annum interest. What sum of money will he receive if he closes the account on 1st August, 2010? [5]

Examiners' Comments

(a) Candidates did not use the given scale i.e. *1cm = 1 unit* along x and y axis. Further the following errors were observed: (i) incorrectly plotted points A *and F*; (ii) Some candidates marked the positive and negative parts of both x and y axis incorrectly; (iii) Some did not join the points in the proper order and hence were unable to name the figure as octagon; (iv) some did not draw or name the line of symmetry.

(b) Many candidates made errors by taking the April balance as ₹2500 instead of '0' as the account was opened on 16th April 2010. Errors were made in calculating the minimum balance and also in finding the amount received on 1st August 2010. Candidates worked out ₹(18350 + 61.17) instead of ₹(8950 +61.17), Some candidates took t = 4 instead of 1.

Suggestions for teachers

- It is necessary to see that students read their question paper and identify the given data. If a scale is given for a question on graph the same must be used. All coordinates must be noted. Students must be instructed to form the figure by joining the points and also draw the line of symmetry and name it.
- Concepts of finding interest using the formula: Interest = (P×R×1)/(100×12) needs to be made clear. Taking t = 1 is important as interest is calculated monthly and must be divided by 12 as rate given is 4% per annum and not per month. Students must be made aware that the answer related to money must be written correctly to 2 decimal places. So interest earned is ₹61.17 and not ₹61.166 or ₹61.2 or ₹61.16

Marking Scheme

Question 5.

(a) (i) Plotting 3 points correctly

(ii) Plotting 3 images correctly

(iii) Writing all coordinates correctly B′(–2,5), C′(–5,2), D′(–5,–2), E′(–2,–5)

(iv) Octagon

(v) y-axis or x-axis or any other correct line of symmetry B_1 x 5 =

(b)

Month	Minimum Balance in ₹
April	0
May	4650
June	4750
July	8950
	18350

$\therefore$ Interest $= \frac{PRT}{100} = \frac{18350 \times 4 \times 1}{100 \times 12} =$ `61.17

Amount received on 1st August = 8950 + 61.17

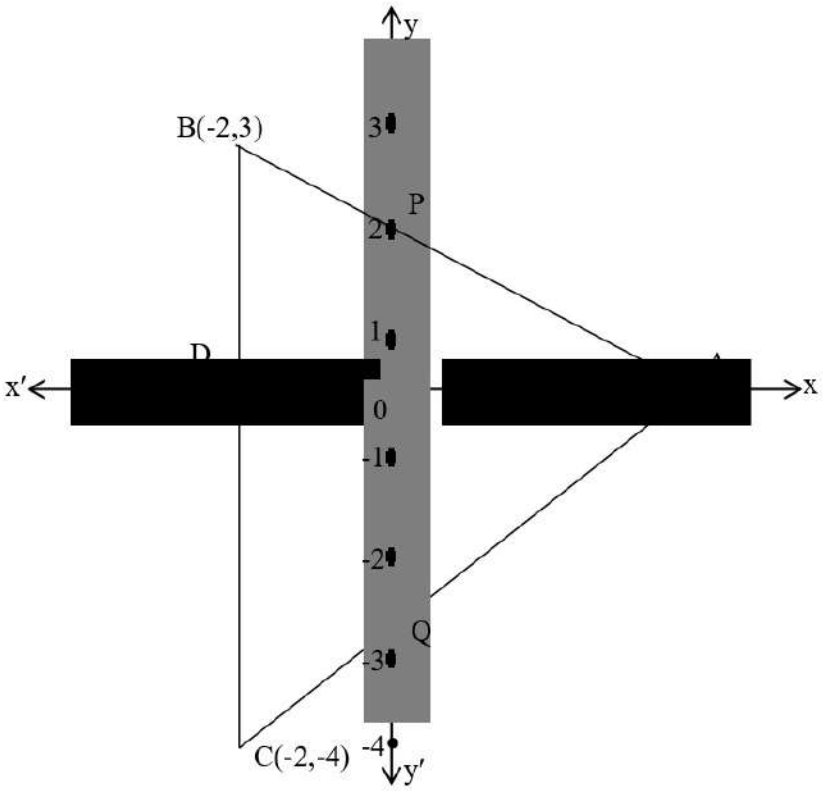
= `9011.17 = `9011

Question 6

(a) If a, b, c are in continued proportion, prove that

$(a + b + c)\ (a - b + c) = a^2 + b^2 + c^2$. [3]

(b) In the given figure ABC is a triangle and BC is parallel to the y-axis. AB and AC intersects the y-axis at P and Q respectively.



(i) Write the coordinates of A.
(ii) Find the length of AB and AC.
(iii) Find the ratio in which Q divides AC.
(iv) Find the equation of the line AC. [4]

(c) Calculate the mean of the following distribution:

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	5	12	35	24	16

[3]

Examiners' Comments

(a) Some candidates failed to apply the property of continued proportion, i.e., $b^2 = ac$ or $\frac{a}{b} = \frac{b}{c} = k$ and hence failed to solve the numerical problem. In some cases, candidates used the property correctly but were unable to simplify and obtain the correct answer.

(b) Some candidates wrote the coordinates of *A as (0,4)* instead of *(4,0)* and hence incorrectly attempted the remaining part of the question. There was an incorrect application of the Distance formula, Section formula and the slope. Hence varied incorrect answers were obtained by candidates.

(c) Most candidates solved the numerical correctly however calculation errors were noticed. Some candidates made errors in finding the class mark and in finding $\sum fx, \sum f$.

Suggestions for teachers

- All properties of ratio and proportion must be given equal importance and a thorough drilling so as to help in handling all types of problems.
- Explain the concept that points on the *x-axis* and *y-axis* must be represented as *(a,0)* and *(0,b)* respectively. It is necessary for a thorough explanation and revision of basic formulae and concepts. Students need to understand that if the question is to find the ratio in which Q divides AC it implies that we are to find $AQ:QC$ and not $CQ:QC$.
- Students must be made to understand all three methods of obtaining Mean and explain how to find the class mark. In using the shortcut or step deviation method students must choose the assumed mean of the class mark for convenience of working.

MARKING SCHEME

Question 6

(a) a,b,c are in continued proportion

$\therefore \frac{a}{b} = \frac{b}{c}$ *or* $b^2 = ac$

LHS $= (a+b+c)(a-b+c) = a^2 + c^2 + 2ac - b^2$

$= a^2 + c^2 + 2b^2 - b^2$

$= a^2 + b^2 + c^2$ = R.H.S. proved

(b) (i) $A \rightarrow (4,0) \rightarrow$

(ii) $AB = \sqrt{(4+2)^2 + (0-3)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

$AC = \sqrt{(4+2)^2 + (0+4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ any one correct

(iii) $\frac{m}{n} = \frac{x-x_1}{x_2-x} = \frac{0-4}{-2-0} = \frac{2}{1}$ i.e. $m:n = 2:1$

(iv) Slope of $AC = \frac{0+4}{4+2} = \frac{4}{6} = \frac{2}{3}$ $\therefore$ eqn. $y - 0 = \frac{2}{3}(x-4)$

Or $2x - 3y = 8$

(c)

Class Interval	Frequency	X	d=X-A	fd
0 – 10	8	5	–30	–240
10 – 20	5	15	–20	–100
20 – 30	12	25	–10	–120
30 – 40	35	35	0	0
40 – 50	24	45	10	240
50 – 60	16	55	20	320
	$\varepsilon f = 100$			$\varepsilon fd = 100$

Mean $= A + \frac{\varepsilon fd}{\varepsilon f} = 35 + \frac{100}{100} = 36$

Question 7

(a) Two solid spheres of radii 2 cm and 4 cm are melted and recast into a cone of height 8 cm. Find the radius of the cone so formed. [3]

(b) Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leaves the same remainder when divided by x+3. [3]

(c) Prove that $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta} = \cos\theta + \sin\theta$ [4]

Examiners' Comments

(a) Candidates made mistakes in writing the formula of the cone and sphere. Some made calculation errors mostly because of using the lengthy method of finding out each volume separately. Workings would be easier if the volumes are equated as $\frac{4}{3}\pi 2^3 + \frac{4}{3}\pi 4^3 = \frac{1}{3}\pi r^2 \times 8$. Hence certain terms common throughout the expression cancel off and a simplified expression is obtained.

(b) Most candidates solved the sum correctly. Some candidates failed to equate $f(-3)$ *and* $g(-3)$ to solve for *'a'* instead wrote $f(-3) = 0, g(-3) = 0$ and tried finding *'a'*. Some wrote $f(3) = g(3)$ and thus obtained an incorrect answer.

Suggestions for teachers

- To avoid long calculations there is a need to guide students of using shorter methods of equating and hence solving of numerical problems.
- More of application based problems are necessary for students to practice.
- Simple tips is necessary for avoiding calculation errors and to save time.

(c) The common drawback noticed was of not making the denominator same before taking the LCM. Other errors detected were in the calculation and formula.

For example $\frac{sin^2\theta}{sin\theta-cos\theta}+\frac{cos^2\theta}{cos\theta-sin\theta}$

$\therefore \frac{sin^2\theta}{sin\theta-cos\theta}-\frac{cos^2\theta}{sin\theta-cos\theta}$. $\therefore LCM\ is\ \sin\theta-\cos\theta$,not $(\sin\theta-\cos\theta)(\cos\theta-\sin\theta)$

MARKING SCHEME

Question 7

(a) Sum of the volumes of the two sphere = volume of the cone

$$\frac{4}{3}\pi\times 2^3+\frac{4}{3}\pi 4^3=\frac{1}{3}\pi r^2\times 8$$

Equating $4\times 8+4\times 64=8r^2 \quad \therefore r^2=\frac{288}{8}=36$

$\therefore r=6cm$

(b) $f(x)=ax^3+3x^2-9 \qquad g(x)=2x^3+4x+a$

Remainder on dividing by $x+3$ is

$f(-3)=a(-3)^3+3\times(-3)^2-9 \qquad g(-3)=2\times(-3)^3+4\times(-3)+a$

$=-27a+27-9 \qquad OR \qquad =-54-12+a$

$\therefore -27a+18 \qquad =-66+a$

(c) $\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}=\cos\theta+\sin\theta$

$$\text{L.H.S}=\frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}=\frac{\sin^2\theta}{\sin\theta-\cos\theta}+\frac{\cos^2\theta}{\cos\theta-\sin\theta}$$

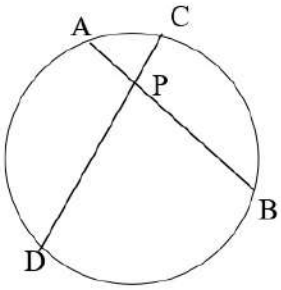
$$=\frac{\sin^2\theta}{\sin\theta-\cos\theta}-\frac{\cos^2\theta}{\sin\theta-\cos\theta}$$

$$=\frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}=\frac{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}{\sin\theta-\cos\theta}=\sin\theta+\cos\theta=\text{R.H.S.}$$

Question 8

(a) AB and CD are two chords of a circle intersecting at P. Prove that

$AP \times PB = CP \times PD$



[3]

(b) A bag contains 5 white balls, 6 red balls and 9 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is:

(i) a green ball

(ii) a white or a red ball

(iii) is neither a green ball nor a white ball. [3]

(c) Rohit invested ` 9,600 on ` 100 shares at ` 20 premium paying 8% dividend. Rohit sold the shares when the price rose to ` 160. He invested the proceeds (excluding dividend) in 10% ` 50 shares at ` 40. Find the:

(i) original number of shares.

(ii) sale proceeds.

(iii) new number of shares.

(iv) change in the two dividends. [4]

Examiners' Comments

(a) In this geometrical proof it was necessary to prove two triangles similar. For this candidates needed to join AD and CD or AC and DB. Hence drawing of diagram was essential. Thus some failed to prove the result. The problem though a circle theorem is actually a direct application of similar triangles, i.e. $\Delta APD \sim \Delta CPB$. Some candidates proved the result but did not give reasons.

(b) Candidates failed to write the favourable outcome and the total instead they directly wrote the required probability. Further, answers were not given in the simplest form, e.g., $\frac{6}{20}$ needed to be written as $\frac{3}{10}$.

Suggestions for teachers

- Emphasise on the following points while solving geometry problems :
- The diagram must always be drawn and labelled carefully; (ii) all reasons supporting the result must be given; (iii) $\angle$APC must not be written as $\angle$ P as there are four angles at that point. Hence supervision is required with regards to naming angles.

(c) Candidates did not seem to be versed with the concept that ₹100 share at ₹20 premium implies that the MV is ₹120.As a result the number of shares was incorrect. Some worked out as ₹9600 ÷ 100 instead of ₹9600 ÷ 120. Some candidates misunderstood the concept of the proceeds, hence committed errors in finding the new number of shares and dividend.

- Revise problems on shares and dividends extensively so that students are familiar with various terms.
- It is necessary to teach students the three basic points of solving a probability sum; (i) identify the total number of outcomes and the number of outcomes favorable for the event; (ii) Finding probability of event E by using P(E)=(No. of favourable outcomes)/ (Total number of outcomes) ;
(iii) Writing of the final answer in the simplest form.

MARKING SCHEME

Question 8

(a) In the two triangles APD and CPB

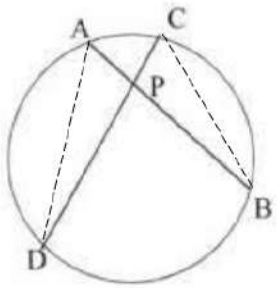
$\angle A = \angle C$ (angles in the same segment)

$\angle D = \angle B$

$\angle APD = \angle CPD$

$\therefore \Delta APD \sim \Delta CPB$ (AAA)

Hence $\frac{AP}{CP} = \frac{PD}{PB}$ or AP x PB = CP x PD



(b) 5 white balls, 6 red balls and 9 green balls

(i) There are 9 green balls and 5 + 6 + 9 = 20 balls altogether

$\therefore$ P (a green ball) = $\frac{9}{20}$

(ii) There are 5 white balls and 6 red balls and their sum is 11

Total number of balls = 20

$\therefore$ P (a white or red ball) = $\frac{11}{20}$

(iii) Neither green or white means probability of a red ball. There are 6 red balls

Total number of balls = 20

$\therefore$ P(neither green nor white) = $\frac{6}{20} = \frac{3}{10}$

(c) Total investment = ₹ 9600 N.V. = ₹100 M.V. = 100 + 20 = ₹120, rate of dividend = 8%.

(i) ∴ original number of shares = $\frac{9600}{120}$ = 80

(ii) Sale proceeds = 80 x 160 = ₹12800

(iii) The new number of shares = 12800 ÷ 40 = 320

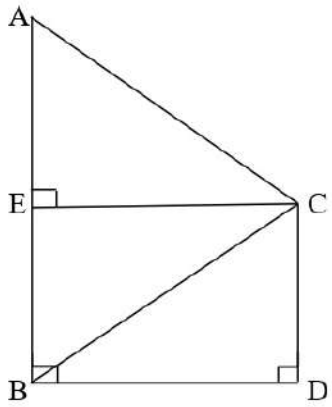
(iv) Dividend from the original shares = 80 x 100 x $\frac{8}{100}$ = ₹640

Dividend from the new shares = 320 x 50 x $\frac{10}{100}$ = ₹1600

∴ Change in the two dividends = ₹1600 – ₹640 = ₹960

Question 9

(a) The horizontal distance between two towers is 120m. The angle of elevation of the top and angle of depression of the bottom of the first tower as observed from the second tower is 30° and 24° respectively.



Find the height of the two towers. Give your answer correct to 3 significant figures. [4]

(b) The weight of 50 workers is given below:

Weight in Kg	50-60	60-70	70-80	80-90	90-100	100-110	110-120
No of Workers	4	7	11	14	6	5	3

Draw an ogive of the given distribution using a graph sheet. Take 2 cm = 10 kg on one axis and 2cm = 5 workers along the other axis. Use a graph to estimate the following:

(i) the upper and lower quartiles.
(ii) if weighing 95 Kg and above is considered overweight find the number of workers who are overweight. [6]

Examiners' Comments

(a) The following errors were observed among different candidates (i) Incorrect value of tan 24° or was unable to find its value at all; (ii) Many did not write their answer correct to 3 significant figures as given in the question; (iii) Some candidates rounded off tan 24° *as* 0.445 *or* 0.44 and $\sqrt{3}$ as 1.73 or 1.7 initially and hence wrote an inaccurate answer.

(b) A few common errors made by candidates in solving this sum are as follows: (i) scale not according to that given in question; (ii) Some made mistakes in finding the cumulative frequency . It is necessary to tally the last *cf i.e.* (50) with $\sum f = 50$ to avoid errors; (iii) Some used a ruler to join the plotted points instead of drawing a free hand curve. A *kink* was not shown between interval 0 and 50 since the next marking is 60; (iv)while using the graph to find median quartiles etc, the perpendicular lines on *x and y-axis* must be drawn; (v) Some candidates interchanged the *x-axis and y-axis.*

Suggestions for teachers

- Advise students to (a)read trigonometric tables;(b) read the question carefully and to specially note whether an approximation of answer is necessary; (c) round off their answers according to the question only at the final step of working so as to get the right answer.
- It must be noted that the ogive is a cumulative frequency curve and the plotted points must be joined freehand and not with a ruler.
- It is necessary to use the given scale and mark the axes accordingly. With intent drilling and supervision such errors may be avoided.

MARKING SCHEME

Question 9

(a) $\tan 24° = \frac{CD}{120}$ *or* $CD = 120 \tan 24$

$= 120 \times 0.4452$

$= 53.424$

$= 53.4\text{m}$

$\tan 30° = \frac{AE}{EC} = \frac{AE}{120}$

$\therefore AE = 120 \times \tan 30 = 120 \times \frac{1}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40 \times 1.732 = 69.28$

$\therefore$ Tower $AB = AE + EB = AE + CD = 69.28 + 53.4 = 122.68$

$= 123\text{m}$

Tower $CD = 53.4\text{m}$ (A)

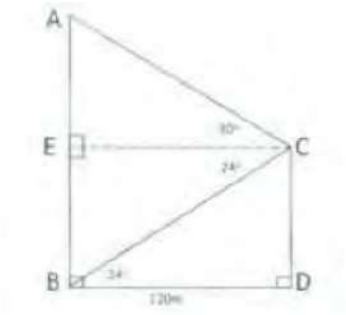
$\tan 24° = \frac{CD}{120}$ *or* $CD = 120 \tan 24$

$= 120 \times 0.4452$

$= 53.424$

$= 53.4\text{m}$

$\tan 30° = \frac{AE}{EC} = \frac{AE}{120}$



$\therefore AE = 120 \times \tan 30 = 120 \times \frac{1}{\sqrt{3}} = \frac{120\sqrt{3}}{3}$

$= 40 \times 1.73 = 69.28$

$\therefore$ Tower $AB = AE + EB = AE + CD = 69.28 + 53.4 = 122.68$

$= 123\text{m}$

Tower CD = 53.4m (A_1)

(b)

Weight in Kg	Number of persons	C.f.
50 – 60	4	4
60 – 70	7	11
70 – 80	11	22
80 – 90	14	36
90 – 100	6	42
100 – 110	5	47
110 – 120	3	50

S-curve plotted with respect to upper boundaries and C.f.

(i) Q_1 position $= \frac{1}{4} \times 50 = 12.5$ $\therefore Q_1 = 71$ kg (± 1)

Q_3 position $= \frac{3}{4} \times 50 = 37.5$ $\therefore Q_3 = 93$ kg (± 1)

Correct axis and perpendiculars dropped for result

(ii) Number of persons who are overweight is equal to

$50 - 38.5 = 11.5\ (\pm 1) = 11$ or 12 approximately

Question 10

(a) A wholesaler buys a TV from the manufacturer for ` 25,000. He marks the price of the TV 20% above his cost price and sells it to a retailer at a 10% discount on the marked price. If the rate of VAT is 8% , find the:

(i) marked price.

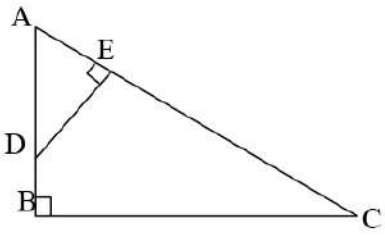
(ii) retailer's cost price inclusive of tax.

(iii) VAT paid by the wholesaler. [3]

(b) If $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

Find AB – 5C. [3]

(c) ABC is a right angled triangle with ∠ABC = 90°. D is any point on AB and DE is perpendicular to AC. Prove that:–



(i) ΔADE ~ ΔACB.

(ii) If AC = 13 cm, BC = 5 cm and AE = 4 cm. Find DE and AD.

(iii) Find, area of ΔADE: area of quadrilateral BCED. [4]

Examiners' Comments

(a) Some candidates calculated 10% discount on ₹25000 instead of the MP which is $\frac{120}{100} \times 25000 = ₹30000$. Hence the VAT found was also incorrect.
A number of calculation errors were also identified. The wholesaler's VAT was incorrect due to lack of understanding.

(b) Some candidates lost marks due to incorrect matrix multiplication. Some made sign errors in finding $-5C$ *where* $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

(c) Few candidates failed to prove $\Delta ADE \sim \Delta ACB$. Some were unable to write the proportional sides $\frac{DE}{BC} = \frac{AD}{AC} = \frac{AE}{AB}$, hence could not get the correct values of DE and AD. Some candidates took the ratio of area $\frac{\Delta ADE}{\Delta ABC} = \frac{AE}{AB}$ instead of $\frac{AE^2}{AB^2}$ hence the area of ΔADE and quadrilateral BCED was incorrect.

Suggestions for teachers

- Students must be advised to read the question carefully and try finding answers to each subpart by working out the sum one step at a time. This helps in avoiding errors.
- Adequate practice is necessary for various matrix operations.
- It is essential to explain similar triangles and explain that corresponding proportional sides are sides opposite to the corresponding equal angles. It is necessary to explain how the ratio of the areas of similar triangles is proportional to the square of the corresponding

MARKING SCHEME

Question 10

(a) (i) Wholesaler's price is `25000

$$\text{M.P.} = 25000 + \frac{20}{100} \times 25000 = \text{`}30{,}000$$

(ii) Retailer's Price $= 30{,}000 - \frac{10}{100} \times 30{,}000 = \text{`}27{,}000$

$\therefore$ Price inclusive of tax $= 27{,}000 + \frac{8}{100} \times 27000 = \text{`}\ (27000 + 2160)$

$= \text{`}29{,}160$

(iii) VAT paid by wholesaler

$= (27000 - 25000) \times \frac{8}{100} = \text{`}160$ (or by taking difference of two taxes)

(b) $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

$$AB - 5C = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0+35 & 6+21 \\ 0+20 & 4+12 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$= \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$= \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$$

Question 11

(a) Sum of two natural numbers is 8 and the difference of their reciprocal is $\frac{2}{15}$. Find the numbers. [3]

(b) Given $\frac{x^3 + 12x}{6x^2 + 8} = \frac{y^3 + 27y}{9y^2 + 27}$. Using componendo and dividendo find x: y. [3]

(c) Construct a triangle ABC with AB = 5.5. cm, AC = 6 cm and $\angle BAC = 105°$. Hence:

(i) Construct the locus of points equidistant from BA and BC.

(ii) Construct the locus of points equidistant from B and C.

(iii) Mark the point which satisfies the above two loci as P. Measure and write the length of PC. [4]

Examiners' Comments

(a) Some candidates could not frame the equation. Some framed it correctly but failed to solve it as they were unable to factorize $2x^2 + 7x - 60 = 0$.

(b) Candidates were unable to apply the property of compodendo and dividendo and hence solved the sum incorrectly. The correct application is $\frac{x^3+12x+6x^2+8}{x^3+12x-6x^2-8} = \frac{y^3+27y+9y^2+27}{y^3+27y-9y^2-27}$; simplify and get the result. Many failed to write the answer in the simplest form, e.g., $\frac{2x}{4} = \frac{2y}{6}$ *is* $\frac{x}{2} = \frac{y}{3} \therefore x:y = 2:3$. Some were unable to identify $x^3 + 12x + 6x^2 + 8 = (x+2)^3$.

(c) In the construction of 105° ruler and compass was not used. The length of AB and AC was incorrect and must be drawn carefully. Some candidates bisected their required line and angle incorrectly and hence were unable to construct the right figure. Correct the answer to the nearest whole number.

Suggestions for teachers

- Emphasis on solving quadratic equation application based problems are necessary.
- Adequate practice of all properties of ratio and proportion is essential. The basic formula for $(a\pm b)^2$,$(a\pm b)^3$ must be recapitulated periodically.
- Instruct students to read the question carefully so that they do not miss any part of the question. The basic two concepts that (i) point equidistant from two arms of an angle lies on the bisector of the angle (ii) a point equidistant from two fixed points lies on the perpendicular bisector of the line segment joining the two fixed points must be explained thoroughly.

MARKING SCHEME

Question 11

a) Let the number be x and $8 - x$

$$\frac{1}{8-x} - \frac{1}{x} = \frac{2}{15}$$

$$\therefore \frac{x-(8-x)}{x(8-x)} = \frac{2}{15}$$

$$\therefore \ (2x - 8) \times 15 = 2(8x - x^2)$$

$$\therefore 30x - 120 = 16x - 2x^2$$

$$2x^2 + 14x - 120 = 0$$

OR $x^2 + 7x - 60 = 0$

$$(x + 12)(x - 5) = 0$$

$\therefore \ x = 5$ $\quad (x = -12$ not possible)

$\therefore$ The number are 3 and 5

b) $\frac{x^3+12x}{6x^2+8} = \frac{y^3+27y}{9y^2+27}$

By componendo & dividendo

$\frac{x^3+12x+6x^2+8}{x^3+12x-6x^2+8} = \frac{y^3+27y+9y^2+27}{y^3+27y-9y^2-27}$

$\frac{(x+2)^3}{(x-2)^3} = \frac{(y+3)^3}{(y-3)^3}$ $\therefore \frac{x+2}{x-2} = \frac{y+3}{y-3}$

$\therefore \frac{x+2+x-2}{x+2-x+2} = \frac{y+3+y-3}{y+3-y+3}$

Hence $\frac{2x}{4} = \frac{2y}{6}$

$\therefore x : y = 2 : 3$

c) $\angle BAC = 105°$

ΔABC

Bisector $\angle B$

Bisector of BC or

$PC = 5$cm

Topics/Concepts Found Difficult

- Value Added Tax (VAT)
- Compound Interest inverse problems.
- Trigonometry
- Similarity
- Rounding off final result e.g. significant figures.
- Theorems on properties of circle.
- Properties of proportion.
- Constructing a circle about a constructed Hexagon.
- Calculation of mean
- Coordinate geometry, Section formula and identifying points on x or y axis.
- Quadratic equation problem
- Inequation: writing solution and representation on number line

Suggestions for Candidates

- Reading time must be utilized to make the right choice of questions and make oneself familiar with all given data
- More practice must be done on rounding off of digits
- Use graph paper for questions based on graphs
- Use of log table to find square root of numbers
- Avoid skipping steps. All necessary steps must be clearly shown
- Working for matrix multiplication is essential
- Rounding off of decimals
- Adopt methods where lesser calculation is necessary to get final result
- Necessary sample space must be written for probability problems.
- Steps of working is necessary in conversion of trigonometric ratios of complementary angles.
- Reasons must be provided for all geometry problems.

ICSE Class 10 Maths Question Paper Solution 2016

MATHEMATICS

Question 1		
(a)	Using remainder theorem, find the value of k if on dividing $2x^3 + 3x^2 - kx + 5$ by $x - 2$, leaves a remainder 7.	[3]
(b)	Given A = $\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A^2 = 9A + mI$. Find m.	[4]
(c)	The mean of following numbers is 68. Find the value of 'x'. 45, 52, 60, x, 69, 70, 26, 81 and 94. Hence estimate the median.	[3]

Comments of Examiners

(a) Candidates made mistakes due to the following reasons:
- (i) Instead of using Remainder and Factor Theorem candidates divided the polynomial by (x – 2).
- (ii) Some candidates equated the remainder to zero instead of equating to 7.
- (iii) Some made errors in substituting x=2. Hence got incorrect value of 'k'

(b) Most candidates failed to differentiate between a constant term and a matrix. With this incorrect concept some candidates considered 'm' to be a 2 by 2 matrix, e.g., $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.
Some candidates seemed to be unaware that division of a matrix by another matrix is not possible and hence divided $\begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$ by $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ to find A. Matrix A^2 was deduced by some candidates by squaring each element of the matrix instead of finding

$$\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$$

(c) Candidates made mistakes in finding $\sum x$ *and* $\sum f$. The common error was in writing $\sum x = 497x$ *instead of* $497+x$; $\sum f = 8$ *instead of* 9.
A number of candidates were unable to calculate the median of the given distribution.

Suggestions for teachers

- ✓ Stress on the basic concepts of remainder theorem. Applying the theorem is essential.
- ✓ Operations of matrix addition, scalar multiplication and multiplication of a matrix by another matrix must be made clear to students. Students must be made aware of the fact that matrix A multiplied by Identity matrix I is equal to A provided they are conformable for multiplication.
- ✓ Revision of calculations of mean, median and mode could help in eliminating such errors.

MARKING SCHEME

Question 1.

(a)

$f(x) = 2x^3 + 3x^2 - kx + 5$

$f(2) = 2 \times 2^3 + 3 \times 2^2 - k \times 2 + 5$ M_1 (Substitution)

$16 + 12 - 2k + 5 = 7$ M_1 (equated to 7)

$33 - 2k = 7$

$\therefore 2k = 33 - 7$

$2k = 26$

$\therefore k = 13$ A_1

(b)

$A^2 = 9A + mI$

$$\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} = 9\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} + m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2\times 2 + 0 \times -1 & 2 \times 0 + 0 \times 7 \\ -1 \times 2 + 7 \times -1 & -1 \times 0 + 7 \times 7 \end{bmatrix} = \begin{bmatrix} 18 & 0 \\ -9 & 63 \end{bmatrix} + \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix} = \begin{bmatrix} 18+m & 0 \\ -9 & 63+m \end{bmatrix}$$ M_1 (Product A^2 or 9A)

$\therefore 18 + m = 4$ M_1

Or $m = -18 + 4$

$m = -14$ A_1

Or

$63 + m = 49$

$\therefore m = 49 - 63$

$m = -14$

(c)

$\bar{x} = 68$ (given)

$$\frac{45 + 52 + 60 + x + 69 + 70 + 26 + 81 + 94}{9} = 68$$ M_1

$$\frac{497 + x}{9} = 68$$

$x = 115$ A_1

26, 45, 52, 60, 69, 70, 81, 94, 115.

Median $= \frac{(9+1)th\ term}{2} = 5th\ term = 69$ B_1

Question 2		
(a)	The slope of a line joining P(6, k) and Q(1-3k, 3) is $\frac{1}{2}$. Find (i) k (ii) Midpoint of PQ, using the value of 'k' found in (i).	[3]
(b)	Without using trigonometrical tables, evaluate: $cosec^2 57° - tan^2 33° + cos 44° cosec 46° - \sqrt{2} cos 45° - tan^2 60°$	[4]
(c)	A certain number of metallic cones, each of radius 2 cm and height 3 cm are melted and recast into a solid sphere of radius 6cm. Find the number of cones.	[3]

Comments of Examiners

(a) Candidates used incorrect formula of slope: $\frac{x_2-x_1}{y_2-y_1}$ instead of $\frac{y_2-y_1}{x_2-x_1}$ and hence were unable to find the correct value of k. Some candidates found $-k = 11$ but failed to identify $k = -11$.

(b) Errors in application of complementary angles is very common among candidates. Some wrote $cosec^2 57° = cosec^2(90° - 57°)$ instead of $cosec^2(90° - 33°)$ or $sec^2(90° - 57°)$ etc. Candidates applied incorrect values for tan 60°, cos 45°. Some applied complementary angles directly without showing any working, hence lost marks.

(c) Many candidates used incorrect formula for volume of cone and sphere. Some made mistakes in calculation.

Suggestions for teachers

- ✓ Revision of different types of sums related with slope, mid-point, distance formula is necessary to overcome such errors. Visual aids could also be helpful for retention of such concepts.
- ✓ Thorough practice of complementary angles and their properties is necessary. Further errors in values of special angles e.g., $0°, 30°, 45° 60° and\ 90°$ may be avoided by teaching them methods to deduce the values. Students must be made aware that all essential steps of working must be shown.
  ***e.g.* cosec 57° *must not be directly written as* sec 33°**
- ✓ Students need additional practice with mensuration formula. To avoid calculation errors students must be encouraged to find the number of cones by forming an equation without finding the volume of cone and sphere separately, e.g., $n = \frac{{}^4/_3 \pi R^3}{\frac{1}{3}\pi r^2 h} = \frac{4R^3}{r^2 h}$

MARKING SCHEME

Question 2.

(a)

Correct substitution in slope formula or mid-point formula →M_1

P(6, k) and Q(1 – 3k, 3)

$$m = \frac{1}{2} = \frac{3-k}{1-3k-6}$$

$$-3k - 5 = 6 - 2k$$

$\therefore$ k = –11 A_1

$\therefore$ Points are (6, –11) and (34, 3)

Midpoint of PQ $= \left(\frac{40}{2}, \frac{-8}{2}\right) = (20, -4)$ →A_1

(b)

$$cosec^2 57^\circ - tan^2 33^\circ + cos44^\circ cosec46^\circ - \sqrt{2}cos45^\circ - \tan^2 60^\circ$$

$$cosec^2(90^\circ - 33^\circ) - tan^2 33^\circ + \cos(90^\circ - 46^\circ)\, cosec46^\circ - \sqrt{2}cos45^\circ - (\sqrt{3})^2$$

Any one complementary angle correct M_1

$$(sec^2 33^\circ - tan^2 33^\circ) + sin46^\circ cosec46^\circ - \sqrt{2}\frac{1}{\sqrt{2}} - 3$$

Any one value correct M_1

1 + 1 – 1 – 3 = –2 A_1

(c)

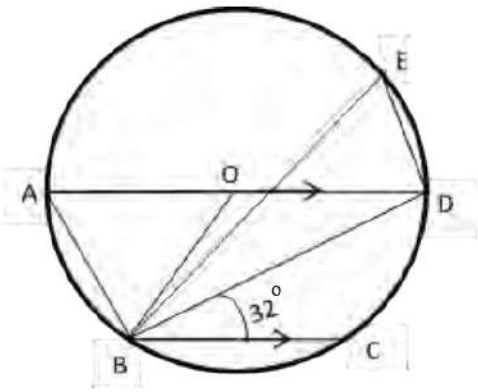
Volume of the 'n' cones = vol. of sphere

$$\therefore n \times \frac{1}{3} \times \pi \times 2^2 \times 3 \qquad M_1$$

$$= \frac{4}{3} \times \pi \times 6^3 \qquad M_1$$

$$\therefore n = \frac{4 \times 6^3}{2^2 \times 3} = 72 \qquad A_1$$

$\therefore$ 72 cones

Question 3		
(a)	Solve the following inequation, write the solution set and represent it on the number line. $$-3(x-7) \geq 15-7x > \frac{x+1}{3}, x \in R$$	[3]
(b)	In the figure given below, AD is a diameter. O is the center of the circle. AD is parallel to BC and $\angle$ CBD = 32^0. Find: (i) $\angle$OBD (ii) $\angle$AOB (iii) $\angle$BED. 	[4]
(c)	**If $(3a+2b):(5a+3b) = 18:29$. Find $a:b$.**	[3]

Comments of Examiners

(a) Some candidates committed errors while transposing like terms on to the same side. Errors were mostly sign error e.g., $-22x > -44 \ \therefore x > 2$ instead of writing $x < 2$. Other common errors were

(i) Candidates did not represent solution in set form;

(ii) Arrow marks not drawn on both ends of the number line

(iii) Number line did not have extra digits marked beyond the solution set;

(iv) -1.5 was not located on the number line

(b) Candidates lost marks in the question for not giving reasons supporting their answers. Some were unable to identify angle ABD to be equal to 90°.

(c) Some candidates were not aware about the basic concepts of ratio and proportion, e.g., $\frac{3a+2b}{5a+3b} = \frac{18}{29}$ was written as $\begin{matrix} 3a+2b=18 \\ 5a+3b=29 \end{matrix}$ which should have been $\begin{matrix} 3a+2b=18k \\ 5a+3b=29k \end{matrix}$ where k is any constant$\neq 0$.

Answer $a:b$ wass written as $\frac{4}{3}$ instead of 4:3.

Suggestions for teachers

- ✓ To get the full credit in solving inequation all the 4 given points are essential and must be followed. Furthermore extensive drilling is necessary related to concepts of positive and negative signs.
- ✓ Thorough practice of all properties of a circle is necessary. Students must be advised to give reasons supporting their results of all geometry problems. Some students need to be guided how to name an angle correctly.
- ✓ Concepts of ratio and proportion needs to be thoroughly explained and must be made clear why it is not possible to equate directly. Students must be advised to read the question carefully and write answers in the form required.

MARKING SCHEME

Question 3.

(a)

$-3x + 21 \geq 15 - 7x$, $\quad 15 - 7x > \frac{x+1}{3}$

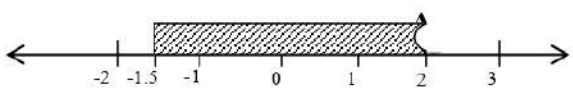
$-3x + 7x \geq 15 - 21$, $\quad 45 - 21x > x + 1$

$4x \geq -6$ $\quad$ **M1** $\quad -21x - x > 1 - 45$

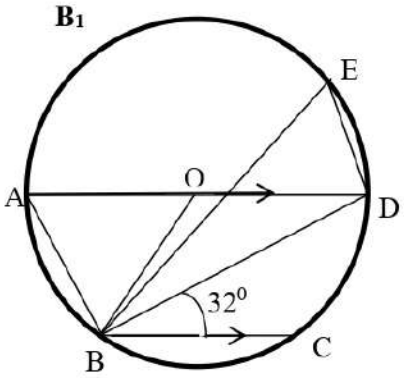
$x \geq -\frac{6}{4}$ $\quad$ (any one) $\quad -22x > -44$

$x \geq -1.5$ $\quad x < 2$

$\{x: -1.5 \leq x < 2, x \in R\}$ $\quad$ **A1**



A_1

(b)	$\angle$ ADB = $\angle$DBC = = 32^0 (alternate interior angles are equal) → **B1** $\angle$OBD = $\angle$ODB = 32^0 ($\because$ OA = OB = OD) **B1** $\angle$AOB= 2 x 32^0 (angle at centre= 2x angle subtended = 64^0 by the same arc at circumference) → **B1** $\angle$ABD = 90^0 (angle in a semicircle is 90^0) $\angle$BAD= 180 – (90 + 32)(sum of the angles in a Δ) = 58^0 **B1** $\angle$BED = $\angle$BAD = 58^0 (angles in the same segment are equal) (ow -1 if at least 2 correct reasons are not written) 
(c)	$\frac{(3a+2b)}{(5a+3b)} = \frac{18}{29}$ $29(3a+2b) = 18(5a+3b)$ **M1** $87a + 58b = 90a + 54b$ $3a = 4b$ **M1** $\therefore a:b = 4:3$ **A1**

Question 4		
(a)	A game of numbers has cards marked with 11, 12, 13, ….. , 40. A card is drawn at random. Find the Probability that the number on the card drawn is: (i) A perfect square (ii) Divisible by 7	[3]
(b)	Use graph paper for this question. (Take 2 cm = 1 unit along both x and y axis.) Plot the points O (0, 0), A (–4, 4), B (–3, 0) and C (0, –3) (i) Reflect points A and B on the y axis and name them A' and B' respectively. Write down their coordinates.	[4]

	(ii) Name the figure OABCB'A'. (iii) State the line of symmetry of this figure.	
(c)	Mr. Lalit invested ₹5000 at a certain rate of interest, compounded annually for two years. At the end of first year it amounts to ₹5325. Calculate (i) The rate of interest. (ii) The amount at the end of second year, to the nearest rupee.	[3]

Comments of Examiners

(a) Common errors of candidates:

(i) They did not list the total outcomes and the favourable outcomes

(ii) some listed the numbers from 1 to 40 whereas it should have been 11 to 40.

(iii) Answer to the result was not written in the simplest form, e.g., $\frac{4}{30}$ was not written as $\frac{2}{15}$.

(b) Some candidates committed errors in plotting points *B and* C which led to an incorrect figure. Some got confused with the images of *A* and *B*. Candidates who did not complete the figure were unable to name it. Figure being irregular some candidates were unable to name the figure correctly and some could not name the line of symmetry.

(c) Most candidates were comfortable with the question and found the rate correctly but failed to write the answer to the nearest Rupee i.e. ₹5671.125 should have been written as ₹5671. Some took ₹ 5325 as amount for two years and hence calculated incorrectly.

Suggestions for teachers

- ✓ Insist on students to list the total and the favourable outcomes. They must be advised to write all answers based on probability in the simplest form.
- ✓ Extensive practice is necessary to plotting points, choosing coordinate axes, reflection and formation of figures of different shapes. Students must be advised to read the question carefully and choose the correct scale as given in the question.
- ✓ Regular revision is necessary for some students so as to correctly identify amount, C.I., They must be made aware of the basic knowledge that in C.I., interest is paid on the interest.

MARKING SCHEME

Question 4.

(a) (i) n (S) = 30

$E_1 = \{16, 25, 36\}$

$P(E1) = \frac{3}{30} = \frac{1}{10}$ **A_1** **B_1 any one outcome correct**

(ii) $E_2 = \{14, 21, 28, 35\}$

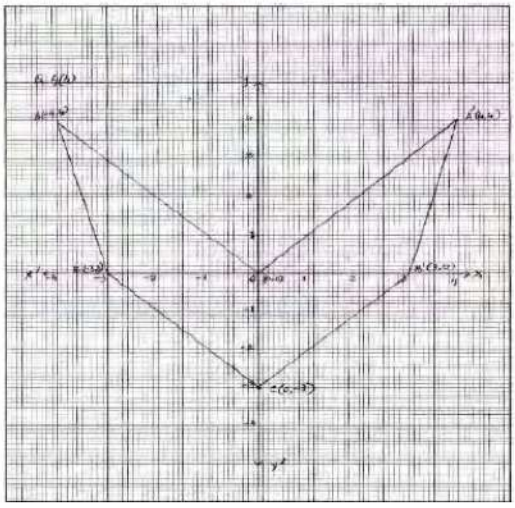
$P(E_2) = \frac{4}{30} = \frac{2}{15}$ **A_1**

(b) Plotting at least 2 points correctly **B_1**

(i) A'(4, 4), B'(3, 0) **B_1 (one correct)**

(ii) Concave Hexagon or Hexagon (completed figure) **B_1**

(iii) y axis or $x = 0$ **B_1**



(c) (i) P = ₹ 5000 and Amount at the end of first year = ₹5325

I = 5325 – 5000 = 325 **M_1** (Applying correctly to find rate)

$R = \frac{325 \times 100}{5000} = 6.5\%$ **A_1**

(ii) C.I for the second year $= \frac{5325 \times 6.5}{100} =$ ₹346.125

Amount at the end of two years = 5325 + 346.125 = 5671.125

$A =$ ₹ 5671 **A_1** (CAO)

SECTION B (40 Marks)

*Attempt any **four** questions from this Section*

Question 5

(a) Solve the quadratic equation $x^2 - 3(x + 3) = 0$; Give your answer correct to two significant figures. [3]

(b) A page from the savings bank account of Mrs. Ravi is given below. [4]

Date	Particulars	Withdrawal (₹)	Deposit (₹)	Balance (₹)
April 3rd 2006	B/F			6000
April 7th	By cash		2300	8300
April 15th	By cheque		3500	11800
May 20th	To self	4200		7600
June 10th	By cash		5800	13400
June 15th	To self	3100		10300
August 13th	By cheque		1000	11300
August 25th	To self	7400		3900
September 6th 2006	By cash		2000	5900

She closed the account on 30th September, 2006. Calculate the interest Mrs. Ravi earned at the end of 30th September, 2006 at 4.5% per annum interest. Hence, find the amount she receives on closing the account.

(c) In what time will Rs.1500 yield Rs.1996.50 as compound interest at 10% per annum compounded annually? [3]

Comments of Examiners

(a) Candidates committed errors in simplifying the equation to $x^2 - 3x - 9 = 0$. Some were unable to find the value of $\sqrt{5}$ and $\sqrt{45}$ hence failed to get the answer.

Most candidates wrote the answer as 4.85 and – 1.85 instead of rounding off to two significant figures and writing the answer as 4.9, -1.9.

(b) Candidates made mistakes in finding minimum balance of various months. Some took September balance as ₹5900 instead of 0. The reason for this is not knowing that balance is zero on a particular month when the account is opened or closed. Candidates wrote answer as 151.5, ₹6051.5, instead of ₹151.50, ₹6051.50.

(c) Answered correctly by most candidates, however a few committed errors in calculations.

Suggestions for teachers

- ✓ Adequate practice is necessary for simplifying quadratic equations and getting in the form $ax^2 + bx + c = 0$ and on approximation of numbers. Encourage students to use logarithm tables to find square roots of numbers.
- ✓ Basic concepts must be made clear to students to avoid such errors. They need to understand the concepts of dividing by 12 to find monthly interest. Concept of writing answers like 151.5 to two decimal places must be made very clear.
- ✓ Students must be advised to solve sums based on C.I., Amount, finding time and rate by simpler methods.

MARKING SCHEME

Question 5.

(a)

$x^2 - 3(x+3) = 0$

$x^2 - 3x - 9 = 0$

$x = \dfrac{-(-3) \pm \sqrt{(9 - 4 \times 1 \times -9)}}{2 \times 1}$ M_1

$x = \dfrac{3 \pm \sqrt{(9 + 36)}}{2} \Rightarrow \dfrac{3 \pm \sqrt{45}}{2} \Rightarrow \dfrac{3 \pm 3\sqrt{5}}{2}$

$x = \dfrac{3 \pm 3 \times 2.236}{2} \Rightarrow \dfrac{3 \pm 6.708}{2}$ M_1

$x = \dfrac{9.708}{2}$, $x = \dfrac{-3.708}{2}$, $x = 4.854$, $x = -1.854$

$x = 4.85$, $x = -1.85$, $x = 4.9$, $x = -1.9$ A_1

(b)

Month	Minimum balance
April	8300
May	7600
June	10300
July	10300
August	3900
Sept.	0
Total P=	40400

M1 (any 3 minimum balance correct)

$$\text{S. I.} = \frac{40400}{} \times \frac{1}{12} \times \frac{4.5}{100} \longrightarrow \textbf{M1}$$

$$= ₹151.50 \longrightarrow \textbf{A1}$$

Amount Mrs. Ravi receives

= ₹ (5900 + 151.50)

= ₹ 6051.50 **A_1**

(c)

$$1996.50 = 1500\left(1+\frac{10}{100}\right)^n$$ M_1 correct sub in correct formula

$$\left(\frac{11}{10}\right)^3 = \left(\frac{11}{10}\right)^n$$ M1 Simplification.

$\therefore n = 3$ years A_1

Question 6

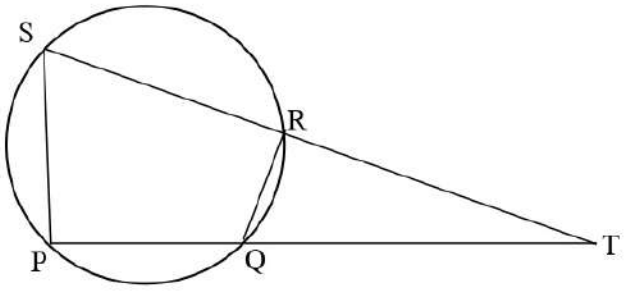
(a) Construct a regular hexagon of side 5 cm. Hence construct all its lines of symmetry and name them. [3]

(b) In the given figure PQRS is a cyclic quadrilateral PQ and SR produced meet at T. [4]

(i) Prove $\Delta TPS \sim \Delta TRQ$.

(ii) Find SP if TP = 18cm, RQ = 4 cm and TR = 6cm.

(iii) Find area of quadrilateral PQRS if area of $\Delta PTS = 27\ cm^2$.



(c)	Given matrix A = $\begin{bmatrix} 4\sin 30° & \cos 0° \\ \cos 0° & 4\sin 30° \end{bmatrix}$ and B = $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$ If AX =B (i) Write the order of matrix X. (ii) Find the matrix 'X'.	[3]

Comments of Examiners

(a) Candidates lost marks in the question for not constructing a hexagon using a ruler and compass. To construct the lines of symmetry candidates failed to use the steps of construction of perpendicular bisector of side and bisector of an angle.

(b) Some candidates were unable to identify two pairs of equal angles so as to prove $\Delta TPQ \sim \Delta TRS$.

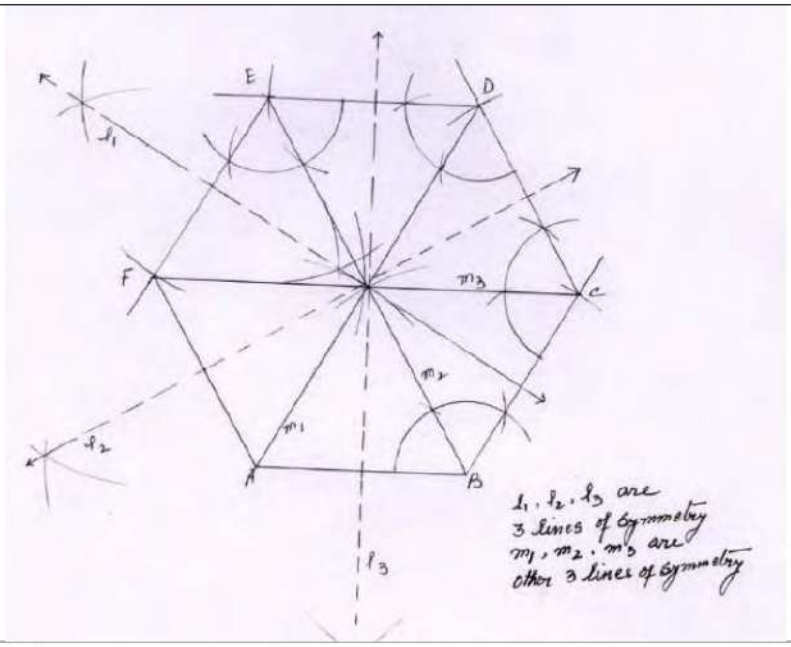
Many candidates managed to write ratio $\frac{SP}{QR} = \frac{TS}{TQ}$ correctly and find $SP=12cm$, but knowledge of ratio of area of similar triangles was not clear, hence they could not find the Area of $Quad\ PQRS$.

(c) Some candidates made mistakes in substituting values of $\sin 30°, \cos 0°$ which led to errors in getting $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Those candidates who got the incorrect value of A could not find the values of a and b.

Suggestions for teachers

- ✓ To avoid errors in construction, insist students to use a ruler and compass with all traces of construction clearly shown.
- ✓ Sufficient practice of similar triangles is necessary for students. They must be taught to write the corresponding proportional sides and area of similar triangles being proportional to the square of the sides i.e. $\frac{\Delta TPS}{\Delta TRQ} = \frac{TP^2}{TR^2}$
- ✓ Students need to have a sound knowledge of standard angles. Concept of identifying order of matrix needs more revision.

MARKING SCHEME **Question 6.**	
(a)	Cosntruction of one side one angle **M1** Hexagon **A1** Lines of symmetry any 2 (one side one angle) **B1**



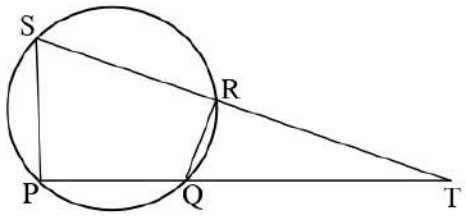
(b) (i) In ΔTPS and ΔTRQ

∠TQP = ∠TSP (ext. angle of a cyclic quad. = interior opposite angle)

∠TRQ = ∠TPS (ext. angle of a cyclic quad. = interior opposite angle)

∠T common to both

∴ ΔTPQ ~ ΔTRQ (AAA) **B_1**



(ii) ΔTPS ~ ΔTRQ

$$\frac{SP}{QR} = \frac{TS}{TQ} = \frac{TP}{TR}$$

Or $\frac{SP}{4} = \frac{18}{6} \therefore SP = \frac{18}{6} \times 4 = 12$ cm **A_1**

(iii) ΔTPS ~ ΔTRQ

$$\frac{\Delta TPS}{\Delta TRQ} = \frac{TP^2}{TR^2} = \frac{18^2}{6^2}$$ **M_1 (any form or directly used)**

$$= \left(\frac{18}{6}\right)^2 = 3^2 = 9$$

$$\frac{27}{\Delta TRQ} = 9$$

$$\therefore \Delta TRQ = \frac{27}{9} = 3\text{cm}^2$$

∴ quad PQRS = 27 – 3 = 24 cm² **A_1**

(c)

(i) $A = \begin{bmatrix} 4\sin 30° & \cos 0° \\ \cos 0° & 4\sin 30° \end{bmatrix}$

$A = \begin{bmatrix} 4 \times \frac{1}{2} & 1 \\ 1 & 4 \times \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$(2 \times 2)(2 \times 1) = (2 \times 1)$

$\therefore$ *order of matri X is* (2×1) **B1**

(ii) Let $X = \begin{bmatrix} a \\ b \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 2a + b \\ a + 2b \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$2a + b = 4$ **M1 Any one method.**

$a + 2b = 5$

$a = 1, b = 2$

$\therefore X = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ **A1**

Question 7

(a) An aeroplane at an altitude of 1500 meters finds that two ships are sailing towards it in the same direction. The angles of depression as observed from the aeroplane are 45° and 30° respectively. Find the distance between the two ships. [4]

(b) The table shows the distribution of the scores obtained by 160 shooters in a shooting competition. Use a graph sheet and draw an ogive for the distribution. (Take 2cm = 10 scores on the X axis and 2cm = 20 shooters on the Y-axis). [6]

Scores	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of shooters	9	13	20	26	30	22	15	10	8	7

Use your graph to estimate the following:

(i) The median.

(ii) The interquartile range.

(iii) The number of shooters who obtained a score of more than 85%.

Comments of Examiners

(a) Most candidates were unable to draw the diagram as per the given data and lost marks. Some candidates made calculation errors while solving the sum. Some took $\sqrt{3} = 1.73$ instead of 1.732; hence final answer turned out to be 1095 instead of 1098.

(b) Candidates made errors in this question at different stages such as:

(i) errors in finding *cf;*

(ii) incorrect scale chosen;

(iii) error in finding Q_1 and Q_3;

(iv) some plotted points with respect to lower boundaries instead of upper boundaries;

(v) some failed to drop perpendiculars to locate median, quartiles etc.

Suggestions for teachers

- ✓ Advise students that all rounding off must be done at the end while calculating the final answer. More drilling is necessary for problems of heights and distances, especially where the diagram needs to be drawn.
- ✓ Students need to be taught the method of cross checking the *cf* found. They must read the question carefully so that they do not miss out the condition given, e.g., scale for graph. Students must also be explained that Ogive is a free hand smooth curve and is not drawn on the graph with a ruler.

MARKING SCHEME

Question 7.

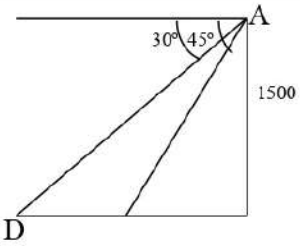
(a)

In ΔABC AB = BC = 1500

$\therefore$ From ΔABD

$\tan 30° = \dfrac{1500}{1500 + x}$ **M_1**

$\therefore \dfrac{1}{\sqrt{3}} = \dfrac{1500}{1500 + x}$



B_1

(correct value of tan30 or tan45) **B_1**

$1500 + x = 1500\sqrt{3}, x = 1500(\sqrt{3} - 1) = 1500 \times (1.732 - 1)$

$= 1500 \times .732 = 1098m$ **A_1**

(b)

c.f: 9,22,42,68, 98, 120, 135, 145, 153, 160 ⟶ **B1** (first 6 correct)

(For smooth S curve plotted with upper boundaries) ⟶ B1

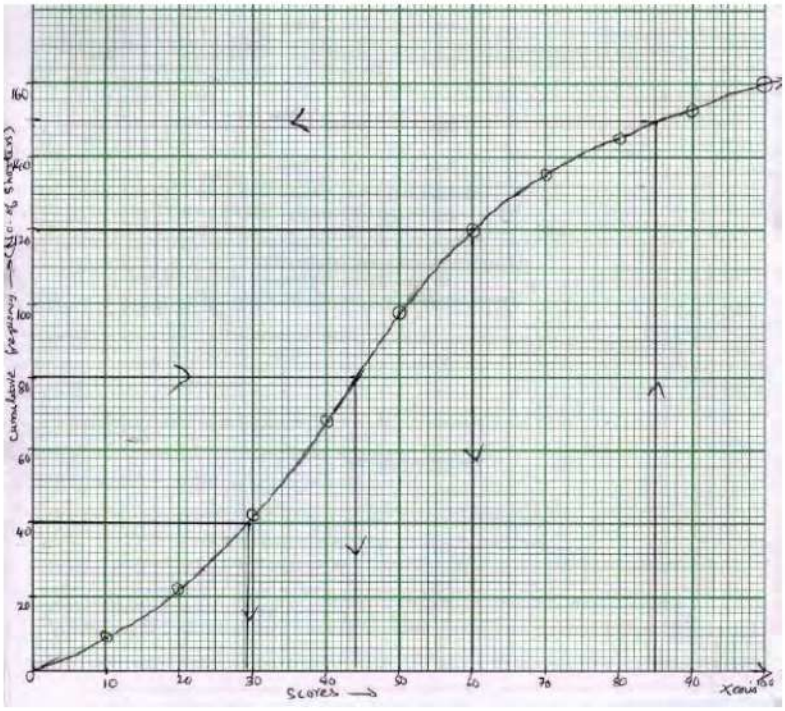
Median = $\dfrac{N^{th}}{2}$ term. = 80th term = 44 (± 1) ⟶ **A1**

$Q_1 = \frac{N}{4}th$ term= 40th term = 29.5 (± 1) ⟵ **A1** (any one correct)

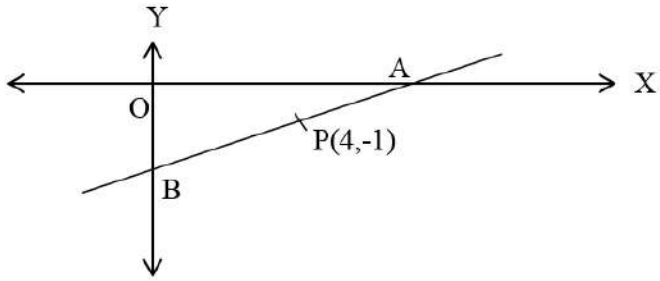
$Q_3 = \frac{3N}{4}th\ term = 120^{th}$ term = 60

Interquartile range = $Q_3 - Q_1$ =60-29.5= 30.5 (± 1) ⟶ **A1**

No. of shooters who obtained a score of more than 85% = 10 ⟶ **A1**



Question 8		
(a)	If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ show that $\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3xyz}{abc}$	[3]
(b)	Draw a line AB = 5 cm. Mark a point C on AB such that AC = 3 cm. Using a ruler and a compass only, construct: (i) A circle of radius 2.5 cm, passing through A and C. (ii) Construct two tangents to the circle from the external point B. Measure and record the length of the tangents.	[4]

(c)	A line AB meets X-axis at A and Y- axis at B. P(4, -1) divides AB in the ratio 1:2. (i) Find the coordinates of A and B. (ii) Find the equation of the line through P and perpendicular to AB.  	[3]

Comments of Examiners

(a) Few candidates were unable to solve the problem due to inadequate concepts of Ratio and Proportion.

Some made mistakes by taking LCM and working with both RHS and LHS simultaneously.

In general, question was solved correctly by most candidates who took $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k$.

(b) Some candidates made errors in locating point *C*. Some committed errors in locating the centre of the circle through *A* and *C*. Candidates were unable to use chord properties as well as tangent properties.

(c) Candidates made errors in applying the correct section formula. Some candidates failed to identify points on coordinate axes *A* and *B* as *(x,0)* and *(0,y)*.

Some candidates found *A* and *B* correctly but were not able to use the perpendicular condition $m_1 \times m_2 = -1$ and hence were unable to find the equation of the line through *P* perpendicular to AB.

Suggestions for teachers

- ✓ Students must be taught that, to prove an identity one must start with either LHS or RHS and not both. simultaneously. Sufficient practice is required to simplify algebraic expression.
- ✓ Expose students to different types of construction based on properties of chord, tangents, loci. Students must be instructed that all traces of construction must be clearly shown.
- ✓ Students require drilling in basic concepts like *y-coordinate* on *x-axis* and *x-coordinate* on *y-axis* is 0. Special attention must be given to find an equation of a line perpendicular to a given line and passing through a give point.

MARKING SCHEME

Question 8.

(a)

Let $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}=k$

$\therefore x=ak, y=bk, z=ck$ **M1**

$LHS\ \frac{a^3k^3}{a^3}+\frac{b^3k^3}{b^3}+\frac{c^3k^3}{c^3}=3k^3$ **M1**

$=3\times\frac{x}{a}\times\frac{y}{b}\times\frac{z}{c}=\frac{3xyz}{abc}=RHS$ **A1**

$RHS=\frac{3\times ak\times bk\times ck}{abc}=3k^3$ (or by taking RHS and proving)

$\therefore LHS=RHS$

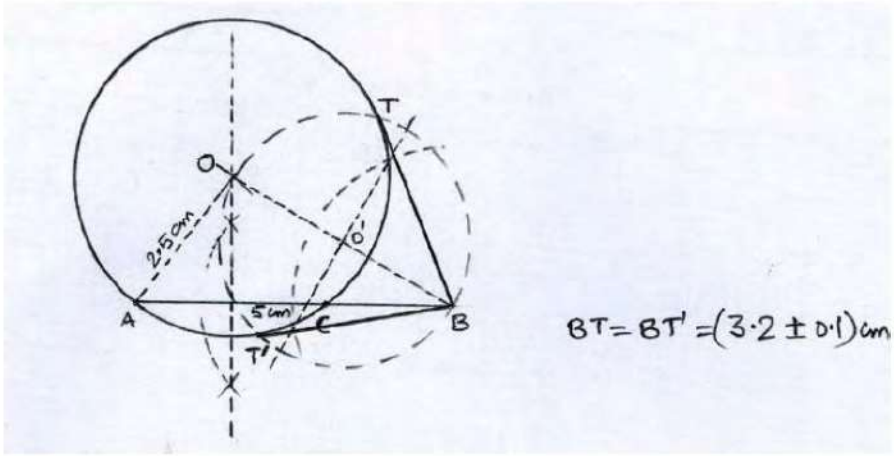
(b)

Construction of perpendicular bisector of AC **M1**

Construction of correct circle **A1**

Construction of arcs/circle on OB as diameter **M1**

BT = BT' = 3.1 cm **A1 ± 1**



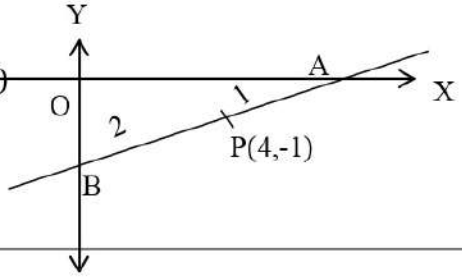
(c)

Let A (x,0) , B(0, y)

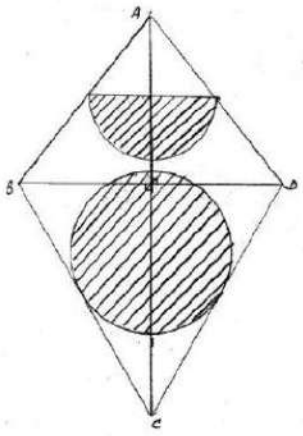
$4=\frac{2x+1\times 0}{2+1}$

$-1=\frac{2\times 0+1\times y}{2+1}$

M1 (any one correct)



	x= 6, y= -3 A(6, 0) , B(0, -3) ⟶ **A1** (any one correct) Slope of AB $= \frac{-3-0}{0-6} = \frac{1}{2}$ Slope of the line ⊥ to AB= -2 Equation of line through P and ⊥ to AB= $y + 1 = -2(x-4)$ $2x + y = 7$ ⟶ **A1**

Question 9		
(a)	A dealer buys an article at a discount of 30% from the wholesaler, the marked price being ₹6,000. The dealer sells it to a shopkeeper at a discount of 10% on the marked price. If the rate of VAT is 6%, find (i) The price paid by the shopkeeper including the tax. (ii) The VAT paid by the dealer.	[3]
(b)	The given figure represents a kite with a circular and a semicircular motifs stuck on it. The radius of circle is 2.5 cm and the semicircle is 2 cm. If diagonals AC and BD are of lengths 12 cm and 8 cm respectively, find the area of the: (i) shaded part. Give your answer correct to the nearest whole number. (ii) unshaded part.  	[4]

(c)	A model of a ship is made to a scale 1 : 300 (i) The length of the model of the ship is 2 m. Calculate the length of the ship. (ii) The area of the deck ship is 180, 000 m^2. Calculate the area of the deck of the model. (iii) The volume of the model is 6.5 m^3. Calculate the volume of the ship.	[3]

Comments of Examiners

(a)Calculation errors was observed. Some candidates found the discounted price but were unable to find the VAT paid by dealer. While finding the net price for the shopkeeper some candidates subtracted ₹324 from 5400 instead of adding the two.

(b) Most candidates managed to find the area of the circle and semi-circle but could not find the area of the kite using $\frac{1}{2}(AC \times DO) + \frac{1}{2}(AC \times BO) = \frac{1}{2}(AC \times BD)$. Some candidates used the incorrect formula for circle hence were incorrect from the initial step. Answer to area of shaded part was not rounded to the nearest whole number as stated in the question.

(c)Some candidates were not clear about proportionality condition and corresponding scale factor k, k^2, k^3 for length, area and volume respectively. Most could find the length but some were unable to find area and volume.

Suggestions for teachers

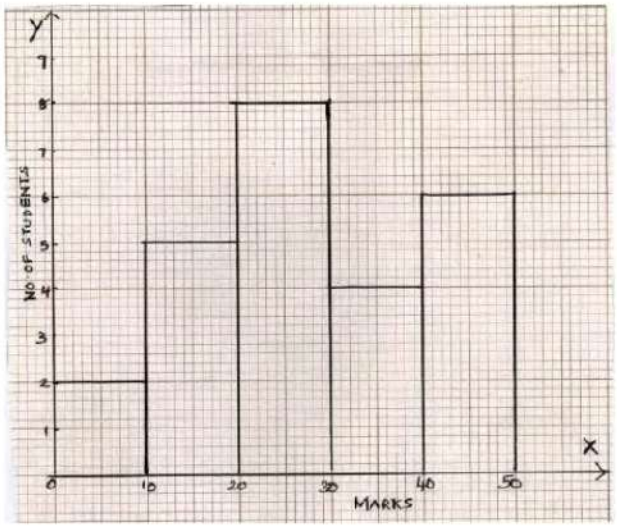
- ✓ It is necessary to practice more sums on VAT so as to be able to identify the requirement of a given question. Due importance must be given to MP, CP, and SP.
- ✓ Students must be made to understand how knowledge of one formula may be used in another. Like applying the area of a triangle to find the area of a Kite. Advise them to use $\pi = \frac{22}{7}$ if not mentioned in the question as it helps in the working. It is essential for students to read questions carefully so as to avoid missing out on important parts.
- ✓ Drilling of problems related to scale factor could help in solving such problems related to models and maps.

MARKING SCHEME

Question 9.

(a)	(i) M R P = ₹ 6000 Cost of article for the dealer $= \frac{70}{100} \times 6000 = ₹\,4200$	$\mathbf{M_1}$ (any one)
	Cost of article for the shopkeeper $= \frac{90}{100} \times 6000 = ₹\,5400$	
	Tax to be paid by the shopkeeper $= \frac{6}{100} \times 5400 = ₹\,324$	
	Net price for the shopkeeper = ₹ (5400 + 324) = ₹ 5724	$\mathbf{A_1}$

	(ii) Value added by the dealer = ₹ (5400 - 4200) = ₹ 1200 $\therefore VAT\ paid\ by\ the\ dealer = 6\%\ of\ ₹1200 = ₹72$ **A1**
(b)	(i) Area of the shaded part $= \pi \times \left(\frac{2^2}{2} + 2.5^2\right)$ **M1 (Any one correct substitution of area)** $= \frac{22}{7}(2 + 6.25)$ $= \frac{22}{7} \times 8.25$ **M1 (finding sum of area)** $= 25.929 = 26\ cm^2$ **A1 (CAO)** (ii) Area of kite $= \frac{1}{2}(AC \times DO) + \frac{1}{2}(AC \times BO) = \frac{1}{2}AC \times BD$ $\frac{1}{2} \times 12 \times 8 = 48$ Area of unshaded part $= 48 - 26 = 22\ cm^2$ **A1**
(c)	$\frac{length\ of\ model}{length\ of\ ship} = \frac{1}{300}$ $\frac{area\ of\ deck\ model}{actual\ area} = (\frac{1}{300})^2$ $\frac{volume\ of\ model}{volume\ of\ ship} = (\frac{1}{300})^3$ Length of ship = 300 x 2 = 600 m ⟶ **B1** Area of deck of model = 180,000 x $(\frac{1}{300})^2$ = 2 m^2 ⟶ **B1** Volume of ship = $(300)^3$ x 6.5 = 175, 500,000 m^3 ⟶ **B1**

Question 10		
(a)	Mohan has a recurring deposit account in a bank for 2 years at 6 % p.a. simple interest. If he gets ₹ 1200 as interest at the time of maturity, find: (i) the monthly instalment (ii) the amount of maturity.	[3]
(b)	The histogram below represents the scores obtained by 25 students in a Mathematics mental test. Use the data to: (i) Frame a frequency distribution table. (ii) To calculate mean. (iii) To determine the Modal class. 	[4]
(c)	A bus covers a distance of 240 km at a uniform speed. Due to heavy rain its speed gets reduced by 10 km/h and as such it takes two hrs longer to cover the total distance. Assuming the uniform speed to be 'x' km/h, form an equation and solve it to evaluate 'x'.	[3]

Comments of Examiners

(a) Some candidates took the monthly instalment of $P = ₹1200$ instead of taking interest as ₹1200 and hence got incorrect results. Some other common errors was in taking time as 2 years or 12 months instead of 24 months.

(b) Many candidates committed calculation errors in finding the mean with some unable to find the class mark. Some made mistakes in finding $\sum fx, \sum f$. Some candidates went on to draw the histogram to find the modal class instead of using the table formed by using the given graph.

(c) Some candidates framed the equation incorrectly $\frac{240}{x-10} - \frac{240}{x} = 2$ was taken as $\frac{240}{x} - \frac{240}{x-10} = 2$. Hence wrote incorrect answers.

Suggestions for teachers

- ✓ Students must be trained to solve more application based problems related to recurring deposit account. Insist that the first and foremost step is to identify the given data.
- ✓ Students must be made clear about class marks of a given distribution. Train them to draw graphs as well as to collect data from a given graph.
- ✓ It is necessary to make the concept very clear that $\frac{240}{x-10} > \frac{240}{x}$ Extensive drilling is necessary for students to formulate quadratic equations

MARKING SCHEME

Question 10.

(a)

$$\text{S.I.} = \frac{p \;\; n \;\; (n+1)}{2} \times \frac{1}{12} \times \frac{R}{100}$$

$$1200 = \frac{p \times 24 \times 25}{2} \times \frac{1}{12} \times \frac{6}{100} \longrightarrow \textbf{M1}$$

Monthly instalment p= ₹800 ⟶ **A1**

Maturity value = p x n + S.I. = 800x24 + 1200 = ₹20400 ⟶ **A1**

(b)

CI	*f*	*x*	*A=25* *D = x – A*	*Fd*
0 – 10	2	5	–20	–40
10 – 20	5	15	–10	–50
20 – 30	8	25	0	0
30 – 40	4	35	10	40
40 – 50	6	45	20	120
	25			70

	Mean = $25 + \frac{70}{25}$ $= 25 + 2.8 = 27.8$ Any 3 CI vs f correct M_1 Substituting in correct formula M_1 (any method) Mean = 27.8 A_1 Modal class 20 – 30 B_1
(c)	Let the uniform speed be 'x' km/h Time taken with uniform speed $= \frac{240}{x}\ hrs$ **B_1 (any one)** Time taken with reduced speed $= \frac{240}{x-10}\ hrs$ Given, $\frac{240}{x-10} - \frac{240}{x} = 2\ hrs$ **M_1** $x^2 - 10x - 1200 = 0$ (x - 40)(x + 30)= 0 x = 40 km/h **A_1**

Question 11		
(a)	Prove that $\frac{\cos A}{1+\sin A} + tanA = secA$.	[3]
(b)	Use ruler and compasses only for the following question. All construction lines and arcs must be clearly shown. (i) Construct a Δ ABC in which BC= 6.5 cm, ∠ABC = 60°, AB= 5cm . (ii) Construct the locus of points at a distance of 3.5 cm from A. (iii) Construct the locus of points equidistant from AC and BC. (iv) Mark 2 points X and Y which are at a distance of 3.5cm from A and also equidistant from AC and BC. Measure XY.	[4]
(c)	Ashok invested ₹ 26,400 on 12%, ₹25 shares of a company. If he receives a dividend of ₹2,475, find the: (i) number of shares he bought (ii) Market value of each share	[3]

Comments of Examiners

(a) Some candidates tried to prove the identity by getting $\tan A$ to the RHS i.e. $\frac{\cos A}{1+\sin A} = \sec A - \tan A$.

Some made mistakes while taking $1 + \sin A$ *as* LCM.

A common error found was in simplification

(b) Basic concept of locus was not known to some candidates. Some were unable to trace out:

(i) locus of points at a distance of 3.5cm from A which is a circle with centre at A and radius 3.5cm;

(ii) locus of the points equidistant from AC and BC being bisector of $\angle C$. Some candidates took incorrect measurements to draw the triangle.

(c) Candidates made calculation errors. Some took ₹ 2475 as sum invested and hence number of shares calculated was incorrect. Some candidates took 25 as MV and tried to get the number of shares by dividing ₹ 26400 by ₹25 instead of ₹825.

Suggestions for teachers

- ✓ Students must be advised not to change the form of a given identity. Adequate practice of identities are necessary to avoid such errors in simplification.
- ✓ Insist on students to draw a rough sketch of diagram indicating all given measurements to avoid errors while constructing. Students must be advised to read the question carefully and do the construction stepwise.
- ✓ Problems related to shares and dividends need special attention and repeated practice so as to be able to solve different types of problems correctly. Students must be instructed to first identify the given data and note them down.

MARKING SCHEME

Question 11.

(a)

$$\frac{\cos A}{1+\sin A} + \tan A = \sec A$$

$$LHS = \frac{\cos A}{1+\sin A} + \frac{\sin A}{\cos A} = \frac{\cos^2 A + \sin A(1+\sin A)}{\cos A(1+\sin A)} \longrightarrow \textbf{M}$$

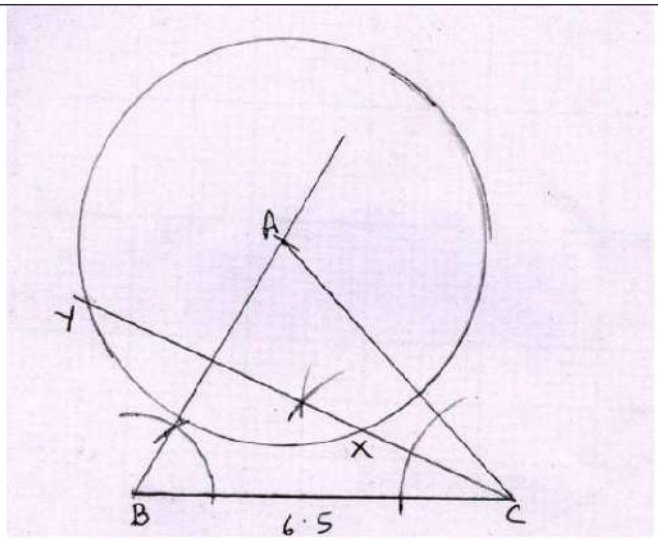
$$= \frac{\cos^2 A + \sin^2 A + \sin A}{\cos A(1+\sin A)}$$

$$= \frac{1+\sin A}{\cos A(1+\sin A)} \quad \textbf{M}_1 \textbf{ (identifying } \sin^2 A + \cos^2 A = 1\textbf{)}$$

$$= \frac{1}{\cos A} = \sec A \longrightarrow \textbf{A1}$$

=RHS

(b)



Construction of Δ ABC ⟶ **B1**

Drawing circle with centre A and radius 3.5 cm ⟶ **M1**

Constructing the bisector of angle C ⟶ **M1**

Locating the points X and Y

XY = 5.2cm ($\pm 0.2cm$) ⟶ **A1**

(c)

Annual income from 1 share = $\frac{12}{100} \times 25 = ₹3$ **M_1**

Total annual income = ₹2475

$\therefore$ Number of shares bought $= \frac{Total\ income}{income\ on\ one\ share} = \frac{2475}{3} = 825$ **A_1**

$\therefore$ Market value of each share $= \frac{Total\ investment}{No.of\ shares} = \frac{26400}{825} = ₹32$ **A_1**

Topics/ Concepts found difficult

(i) VAT

(ii) Shares and Dividend

(iii) Geometry solving problems using properties of circle and similar triangles

(iv) Geometry Constructions

(v) Coordinate geometry: Section formula

(vi) Trigonometry, complementary angles and Heights and Distances.

(vii) Properties of Ratio and Proportion.

(viii) Approximation: to given significant figures or to nearest whole number.

Suggestions for Students:

- Reading time must be utilised to make the right choice of question and to be thorough with the given data.
- More practice is necessary in rounding off of numbers.
- Must choose the correct scale while drawing graphs and special care must be taken while marking the axes and plotting points.
- Logarithm tables may be used to find square roots.
- All steps of working including rough works must be clearly shown on same answer page.
- While solving geometry problems reasons must be given.
- All traces of constructions must be clearly shown.

ICSE Class 10 Maths Question Paper Solution 2018

SECTION A (40 Marks)

*Attempt **all** questions from this Section.*

Question 1

(a) Find the value of 'x' and 'y' if: [3]

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

(b) Sonia had a recurring deposit account in a bank and deposited ₹600 per month for 2½ years. If the rate of interest was 10% p.a., find the maturity value of this account. [3]

(c) Cards bearing numbers 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card which is: [4]

(i) a prime number.

(ii) a number divisible by 4.

(iii) a number that is a multiple of 6.

(iv) an odd number.

Comments of Examiners

(a) Some candidates made mistakes in:
- scalar multiplication of the matrix e.g., $2(y-5)$ was written as $2y-5$ instead of $2y-10$.
- addition and equating corresponding elements of two matrices.

(b) A number of candidates made errors in determining the qualifying principal to find the interest and in finding the maturity value. Some candidates took 'n' as $2\frac{1}{2}$ years instead of 30 months. In calculating interest, some candidates took monthly installment as 60 x 30 = ₹18000 instead of ₹600. For finding matured value, some candidates added ₹600 and the interest.

(c) Many candidates considered the total outcomes as 20 instead of 10. Common errors made were:
(i) Some candidates did not consider '2' as a prime number.
(ii) In identifying multiples of 6, some left out 6.
(iii) Answers were not expressed in the simplest form.
(iv) $\frac{0}{10}$ was not expressed as 0.

Suggestions for teachers

- Give thorough practice of basic operations like addition, multiplication of matrices and solving matrix equation.
- Drill students with basic concepts like monthly installment, qualifying principal to find interest, and maturity value.
- Advise students to read the questions carefully. In such questions, ask them to first identify the total outcomes of the problem.
- Instruct students to express answers in the simplest form.
- Drill students with basic concepts like prime numbers, multiples of numbers, etc.

MARKING SCHEME

Question 1

(a)

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 2x & 14 \\ 18 & 2y-10 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 2x+6 & 14-7 \\ 18+4 & 2y-10+5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 2x+6 & 7 \\ 22 & 2y-5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$\therefore 2x+6=10$ ***OR*** $2y-5=15$

$\therefore 2x=10-6$ $\quad 2y=15+5$

$2x=4$ ***OR*** $\quad 2y=20$

$\therefore x=2$ $\quad \therefore y=10$

(b) Qualifying Sum = $\frac{600\ x\ 30\ x\ (30+1)}{2}$

	$Interest = \frac{600 \text{ x } 30 \text{ x } 31 \text{ x } 10}{2 \text{ x } 12 \text{ x } 100} = ₹2325$ $M.V. = 600 \text{ x } 30 + 2325 = ₹20325$
(c)	2, 4, 6, 8, 10, 12, 14, 16, 18, 20 Total number of cards are 10 (i) Probability of getting a prime number is $\frac{1}{10}$ (ii) Probability of getting a number divisible by 4: Numbers are 4, 8, 12, 16 and 20 $\therefore$ Probability $= \frac{5}{10} = \frac{1}{2}$ (iii) Multiples of 6 are 6, 12, 18. $\therefore$ Probability $= \frac{3}{10}$ (iv) No cards bear an odd number. $\therefore$ Probability $= \frac{0}{10} = 0$

Question 2

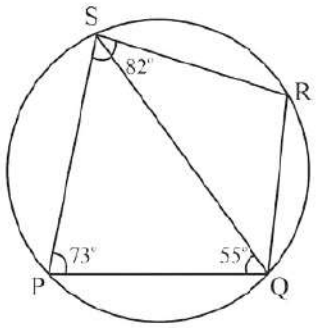
(a) The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. Find the [3]

(i) radius of the cylinder

(ii) volume of cylinder. (use $\pi = \frac{22}{7}$)

(b) If $(k - 3)$, $(2k + 1)$ and $(4k + 3)$ are three consecutive terms of an A.P., find the value of k. [3]

(c) PQRS is a cyclic quadrilateral. Given $\angle QPS = 73°$, $\angle PQS = 55°$ and $\angle PSR = 82°$, calculate: [4]



(i) $\angle QRS$

(ii) $\angle RQS$

(iii) $\angle PRQ$

Comments of Examiners

(a) Some candidates applied incorrect formulas for finding out the radius of the cylinder, i.e., the circumference of the base of the cylinder was taken as πr^2 or $2\pi rh$ instead of $2\pi r$. Also, some used incorrect formula for finding the volume of the cylinder.

(b) The concept of Arithmetic Progression was not clear to many candidates. Many candidates were unable to identify the common difference 'd' hence, failed to equate the difference between second term and first term i.e., $t_2 - t_1$ and third term and second term i.e., $t_3 - t_2$ to evaluate 'k'. Some candidates equated second term of A.P. equal to the sum of third term and the first term i.e., $t_2 = t_3 + t_1$.

(c) Common errors identified in many scripts were as follows:

(i)Properties of circles like, angles in the same segment are equal, opposites angles of a cyclic quadrilateral are supplementary, etc. were not applied correctly.

(ii)Appropriate reasons supporting the answers were missing.

Suggestions for teachers

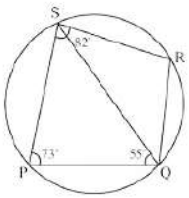
- Give adequate practice for application of various mensuration formulae to prevent students from getting confused between the formulae of volume and surface area of solids.
- Build the concept of series with various examples followed by generalizing the first term of the series, the rule governing the consecutive terms of the series. Thus, develop the common difference of an A.P. and the summation of a given number of terms of the series.
- Emphasise on giving reasons supporting each answer while solving geometry problems. Drill students in naming angles correctly.

MARKING SCHEME

Question 2

(a)	$2\pi r = 132$ $r = 21cm$ $V = \pi r^2 h = \frac{22.21.21.25}{7} = 34650\ cm^3$
(b)	$(k-3), (2k+1), (4k+3)$ $k-3, 2k+1, 4k+3$ in AP implies $(2k+1) - (k-3) = (4k+3) - (2k+1)$ $\therefore\ 2k+1-k+3 = 4k+3-2k-1$ $k+4 = 2k+2$ $\therefore\ 2k-k = 4-2\ or\ k = 2$

(c)



(i) $\angle QRS = 180^\circ - 73^\circ = 107^\circ$ (opposite angles of a cyclic quadrilateral are supplementary)

(ii) $\angle RQS = 180 - (82^\circ + 55^\circ)$

$= 180 - 137$

$= 43^\circ$

(iii) $\angle PSQ = 180^\circ - (73^\circ + 55^\circ)$

$= 52^\circ$

$\therefore \angle PRQ = \angle PSQ = 52^\circ$ ($\angle$s in the same segment are equal)

Question 3

(a) If $(x + 2)$ and $(x + 3)$ are factors of $x^3 + ax + b$, find the values of 'a' and 'b'. [3]

(b) Prove that $\sqrt{sec^2\theta + cosec^2\theta} = tan\theta + cot\theta$ [3]

(c) Using a graph paper draw a histogram for the given distribution showing the number of runs scored by 50 batsmen. Estimate the mode of the data: [4]

Runs scored	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000
No. of batsmen	4	18	9	6	7	2	4

Comments of Examiners

(a) Some candidates made mistakes in substituting x = - 2 or x = - 3 in solving the two equations formed to find ***a*** and ***b***. A few candidates substituted the factors of the given polynomial correctly but failed to equate them to zero at the time of calculation.

(b) In solving identities, some candidates squared the identity either on one side or both sides before proceeding for proving the identity. Many candidates were unable to use identities like $\sec^2\theta = 1 + \tan^2\theta$ etc. Some solved the two sides of the identity simultaneously or interchanged terms from one side to the other.

(c) Some common errors related to the problem were as follows:
- Incorrect choice of scale.
- As the data (runs scored), was from 3000 – 4000, some candidates did not put a kink to explain the gap between origin and the first number.
- Bars drawn by a few candidates were not of equal width.
- Some candidates plotted the graph with cumulative frequency instead of frequency.
- Method of identification of Mode from graph was incorrect.
- Many candidates took the reading from the graph as 4006 instead of 4600.

Suggestions for teachers

- Insist that students should strictly follow the instructions given in all questions.
- Give ample practice to solve the problems based on Remainder and Factor Theorem.
- Give sufficient revision of simultaneous equations to the students so that they can apply the concept in other topics wherever it is required.
- Advise students to prove identities with one side at a time instead of working with both sides together. Adequate drilling is essential with trigonometric identities.
- Clarify the rules of plotting graphs thoroughly to the students. Method to locate mode need sufficient practice.

MARKING SCHEME

Question 3

(a) $(x + 2)$ and $(x + 3)$ factors → $x^3 + ax + b$

$f(-2) = (-2)^3 + a(-2) + b$ $\qquad$ $f(-3) = (-3)^3 + a(-3) + b$

$\therefore 0 = -8 - 2a + b \Rightarrow -2a + b = 8$ $\qquad$ $0 = -27 - 3a + b \Rightarrow -3a + b = 27$

$-2a + b = 8$

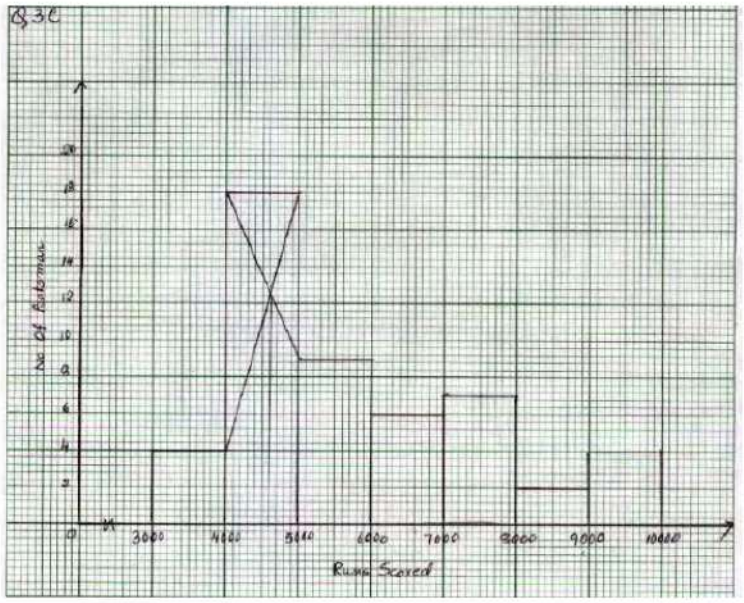
$-3a + b = 27$

$+ \quad - \quad -$

$a = -19$

$38 + b = 8$

$b = -30$

(b)	$LHS = \sqrt{sec^2\theta + cosec^2\theta} = \sqrt{1 + tan^2\theta + 1 + cot^2\theta}$ $= \sqrt{tan^2\theta + cot^2\theta + 2} = \sqrt{(tan\theta + cot\theta)^2} = tan\theta + cot\theta = RHS$ (Using any one identity correctly)
(c)	Mode = 4600 runs Axis and Scale: On X axis: 2 cm = 1000 runs On Y axis: 2 cm = 4 batsmen 

Question 4

(a) Solve the following inequation, write down the solution set and represent it on the real number line: [3]

$$-2 + 10x \leq 13x + 10 < 24 + 10x, x \in Z$$

(b) If the straight lines $3x - 5y = 7$ and $4x + ay + 9 = 0$ are perpendicular to one another, find the value of a. [3]

(c) Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places. [4]

Comments of Examiners

(a) Many candidates made errors in transposing like terms on same side. Some candidates made mistakes in solving the inequation. Most of the errors were pertaining to positive and negative signs. A number of candidates did not write down the solution set after solving the given inequation. Representation on number line was incorrect due to the following reasons: -

$-3x \leq 12$ was simplified and written as $3x \leq -12$ instead of $3x \geq -12$. Some candidates took $x \in R$ instead of $x \in Z$; others drew the number line without arrows. A few candidates failed to put extra number on each side of solution for indicating the continuity of the number line.

(b) Many candidates could not find the correct slope of the two lines. Some candidates used the perpendicular condition $m_1 m_2 = -1$ incorrectly. Quite a few made calculation errors.

(c) Common errors made by candidates were:

- use of incorrect formula.
- mistakes in arranging the equation in standard quadratic equation form.
- mistakes in calculation.
- errors in finding square root of $\sqrt{77}$.
- not writing the answer correct up to two decimal places or not rounding off at all.

Suggestions for teachers

- Give thorough practice on using positive and negative signs.
- Drill thoroughly in writing the solution set after solving the inequation and with representation of the same on the number line. Familiarise students with the symbols *Z, N, I, W* and *R*.
- Give adequate practice in finding the slope of a line from a given equation. Also drill students about the use of conditions for slope of two parallel lines/ perpendicular lines.
- Encourage students to use Mathematical tables to find square roots of numbers.
- Rigorous drilling is necessary on approximation so that candidates are able to round off as required in the question.

MARKING SCHEME

Question 4

(a)

$-2 + 10x \leq 13x + 10 < 24 + 10x$

$-2 + 10x \leq 13x + 10$ $\qquad$ $13x + 10 < 24 + 10x$

$10x - 13x \leq 10 + 2$ $\qquad$ $13x - 10x < 24 - 10$

$-3x \leq 12$ $\qquad$ *or* $\qquad$ $3x < 14$

$3x \geq -12$ $\qquad$ $x < \frac{14}{3}$

$\therefore x \geq -4$ $\qquad$ $x < 4\frac{2}{3}$

Solution: {-4, -3, -2, -1, 0, 1, 2, 3, 4}

(b)

$3x - 5y = 7 \Rightarrow y = \frac{3}{5}x - \frac{7}{5}$ $\quad \therefore slope\ m_1 = \frac{3}{5}$

$4x + ay + 9 = 0 \Rightarrow y = -\frac{4}{a}x - \frac{9}{a}$

$\therefore\ slope\ m_2 = -\frac{4}{a}$

$\therefore \left(\frac{3}{5}\right)\left(-\frac{4}{a}\right) = -1 \Rightarrow a = \frac{12}{5}$

(c)

$x^2 + 7x = 7 \Rightarrow x^2 + 7x - 7 = 0$

$a = 1, b = 7, c = -7$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{(-7)^2 - 4 \times 1 \times (-7)}}{2 \times 1}$

$x = \frac{-7 \pm \sqrt{49 + 28}}{2 \times 1} = \frac{-7 \pm \sqrt{77}}{2}$

$= \frac{-7 \pm 8.775}{2}$

$= \frac{-7 + 8.775}{2}, \quad \frac{-7 - 8.775}{2}$

$\frac{1.775}{2}, \quad \frac{-15.775}{2}$

$= 0.8875, \quad -7.8875$

$x = \{0.89, -7.89\}$

SECTION B (40 Marks)

*Attempt any **four** questions from this Section*

Question 5

(a) The 4^{th} term of a G.P. is 16 and the 7^{th} term is 128. Find the first term and common ratio of the series. [3]

(b) A man invests ₹22,500 in ₹50 shares available at 10% discount. If the dividend paid by the company is 12%, calculate: [3]

(i) The number of shares purchased

(ii) The annual dividend received.

(iii) The rate of return he gets on his investment. Give your answer correct to the nearest whole number.

(c) Use graph paper for this question (Take 2 cm = 1unit along both x and y axis). ABCD is a quadrilateral whose vertices are A (2,2), B (2, –2), C (0,–1) and D(0,1). [4]

(i) Reflect quadrilateral ABCD on the y-axis and name it as A'B'CD.

(ii) Write down the coordinates of A' and B'.

(iii) Name two points which are invariant under the above reflection.

(iv) Name the polygon A'B'CD.

Comments of Examiners

(a) Several candidates went wrong in calculation. Some candidates tried solving the problem as an A.P. instead of G.P. A few candidates took $\sqrt[3]{8}$ as +2 and -2 instead of only +2. Some candidates tried the sum by trial method by assuming a =2 and r =2.

(b) Many candidates calculated the discounted market value incorrectly hence, annual dividend calculated was also incorrect.

A number of candidates did not round off the yield percent to the nearest whole number as it was asked in the question.

(c) Common errors observed were:

- labelling /choice of axes was incorrect;
- plotting the points A′ and B′ was correct but their coordinates were not written;
- plotting of points (1,0) and (0,1) was incorrect;
- the figure was named as *trapezium* instead of *isosceles trapezium*.

Suggestions for teachers

- Instruct students to read the question heedfully and analyze the given conditions before solving the problem. Give them the clear understanding of the concept of common difference and common ratio so as to avoid mixing up sums on A.P. and G.P.
- Stress upon solving sums using given conditions and not by trial method.
- Ensure that students understand the meaning of the terms *shares at par, at premium, at a discount, nominal value(N.V.), market value(M.V.) and dividend*, etc. clearly. The concept of approximation of result needs thorough drilling.
- Instruct students strictly that after plotting necessary points, their coordinates must also be written. The figure formed must be completed. Names of basic geometrical figures need to be revised. Give students clear idea of invariant points. Drilling of plotting points on x-axis and y-axis is essential.

MARKING SCHEME

Question 5

(a) Let a be the first term and r the common ratio and $T_4 = 16$, $T_7 = 128$

$\therefore a r^{4-1} = 16$ and $a r^{7-1} = 128$

$a r^3 = 16$ $\qquad a r^6 = 128$

$$\frac{a r^6}{a r^3} = \frac{128}{16} \qquad \therefore r^3 = 8 = 2^3$$

$\therefore r = 2$

$ar^3 = 16 \Rightarrow a \times 2^3 = 16$

$\therefore a = 2$

(b) NV = 50

MV= 50 – 10% of 50= 45

(i) Number of shares $= \frac{22500}{45} = 500$

(ii) Annual dividend $= \frac{12x50x500}{100} = 3000$

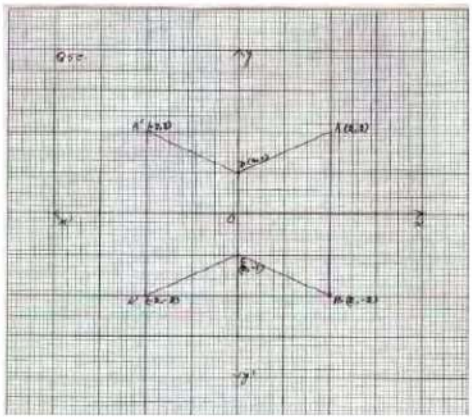
(iii) Yield% $= \frac{3000}{22500} \times 100 = 13.333\% \approx 13\%$

(c) (i) Quadrilateral A'B'CD marked on graph sheet

(ii) A'(–2, 2) and B'(–2, –2)

(iii) C and D

(iv) Isosceles trapezium



Question 6

(a) Using properties of proportion, solve for x. Given that x is positive: [3]

$$\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}}=4$$

(b) $If\ A=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}, B=\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}\ and\ C=\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$, find $AC + B^2 - 10C$. [3]

(c) Prove that $(1+\cot\theta - cosec\theta)(1+tan\theta+sec\theta)=2$ [4]

Comments of Examiners

(a) Many candidates solved the problem without using properties of proportion. Some candidates applied componendo and dividendo but made mistakes in squaring both sides. Quite a few candidates did not follow the instruction for expressing the value of x. They expressed x as $\pm\frac{5}{8}$ instead of $\frac{5}{8}$.

(b) Some candidates found B^2 by squaring each element of matrix B instead of finding matrix product B X B. A few candidates made mistakes in finding the scalar multiplication −10 C.

(c) A number of candidates made mistakes while multiplying the two expressions on the Left-Hand Side of the question. A few candidates worked upto $(1+\frac{cos\theta}{sin\theta}-\frac{1}{sin\theta})(1+\frac{sin\theta}{cos\theta}-\frac{1}{cos\theta})$.
but could not take LCM and simplify. Some were unable to simplify by applying algebraic identity $(a + b)(a - b) = a^2 - b^2$, hence, could not get the correct answer.

Suggestions for teachers

- Advise students to read the question meticulously to avoid errors such as not using properties of proportion.
- Drill previously learnt concepts of algebraic addition, multiplication and applying algebraic identities to square expressions.
- Give sufficient practice in application of properties of proportion.
- Give thorough drilling exercises with matrix multiplication. Stress upon finding square of a matrix. Basic operations with matrix addition and scalar multiplication need repeated practice.
- Give ample practice on basic algebraic operations and identities to enable students to simplify trigonometric identities.
- Revise basic trigonometry formulae on a regular basis in the class.

MARKING SCHEME

Question 6

(a)

$$\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}} = 4$$

By using the property Componendo & Dividendo

$$\frac{2x+\sqrt{4x^2-1}+2x-\sqrt{4x^2-1}}{2x+\sqrt{4x^2-1}-2x-\sqrt{4x^2-1}} = \frac{4+1}{4-1}$$

$$\Rightarrow \frac{4x}{2\sqrt{4x^2-1}} = \frac{5}{3} \Rightarrow \frac{2x}{\sqrt{4x^2-1}} = \frac{5}{3}$$

Squaring both sides

$$\frac{4x^2}{4x^2-1} = \frac{25}{9}$$

$$\Rightarrow 36x^2 = 100x^2 - 25$$

$$\Rightarrow 65x^2 = 25 \Rightarrow x^2 = \frac{25}{64} \Rightarrow x = \pm\frac{5}{8} \therefore x = \frac{5}{8}$$

(b)

$$AC + B^2 - 5C = \begin{bmatrix}2 & 3\\5 & 7\end{bmatrix}\begin{bmatrix}1 & 0\\-1 & 4\end{bmatrix} + \begin{bmatrix}0 & 4\\-1 & 7\end{bmatrix}\begin{bmatrix}0 & 4\\-1 & 7\end{bmatrix} - 10\begin{bmatrix}1 & 0\\-1 & 4\end{bmatrix}$$

$$= \begin{bmatrix}2-3 & 0+12\\5-7 & 0+28\end{bmatrix} + \begin{bmatrix}0-4 & 0+28\\0-7 & -4+49\end{bmatrix} - \begin{bmatrix}10 & 0\\-10 & 40\end{bmatrix}$$

$$= \begin{bmatrix}-1 & 12\\-2 & 28\end{bmatrix} + \begin{bmatrix}-4 & 28\\-7 & 45\end{bmatrix} - \begin{bmatrix}10 & 0\\-10 & 40\end{bmatrix} = \begin{bmatrix}-5 & 40\\-9 & 73\end{bmatrix} - \begin{bmatrix}10 & 0\\-10 & 40\end{bmatrix} = \begin{bmatrix}-15 & 40\\1 & 33\end{bmatrix}$$

(c)

$$(1+cot\theta-cosec\theta)(1+tan\theta+sec\theta) = 2$$

$$\left(\frac{1}{1}+\frac{cos\theta}{sin\theta}-\frac{1}{sin\theta}\right)\left(\frac{1}{1}+\frac{sin\theta}{cos\theta}+\frac{1}{cos\theta}\right)$$

$$\left(\frac{sin\theta+cos\theta-1}{sin\theta}\right)\left(\frac{cos\theta+sin\theta+1}{cos\theta}\right) \qquad (a+b)(a-b) = a^2-b^2$$

OR

$$= \frac{(sin\theta+cos\theta)^2-(1)^2}{sin\theta cos\theta} = \frac{sin^2\theta+cos^2\theta+2sin\theta cos\theta-1}{sin\theta cos\theta}$$

$$= \frac{1+2sin\theta cos\theta-1}{sin\theta cos\theta} = \frac{2\cancel{sin\theta cos\theta}}{\cancel{sin\theta cos\theta}} = 2$$

Question 7

(a) Find the value of k for which the following equation has equal roots. [3]

$$x^2 + 4kx + (k^2 - k + 2) = 0$$

(b) On a map drawn to a scale of 1:50,000, a rectangular plot of land ABCD has the following dimensions. AB = 6 cm; BC = 8 cm and all angles are right angles. Find: [3]

(i) the actual length of the diagonal distance AC of the plot in km.

(ii) the actual area of the plot in square km.

(c) A (2, 5), B (–1, 2) and C (5, 8) are the vertices of a triangle ABC, 'M' is a point on AB such that AM: MB = 1: 2. Find the co-ordinates of 'M'. Hence find the equation of the line passing through the points C and M. [4]

Comments of Examiners

(a) Some candidates were not clear about the nature of roots of Quadratic Equation. Majority of candidates found the discriminant $(4k)^2 - 4(k^2 - k + 2)$ correctly but did not equate it to 0 as per the condition of equal roots. Some candidates took x along with coefficient to find discriminant. A few candidates made errors in solving the equation $12k^2 + k - 8 = 0$ to find the value of k.

(b) Many candidates were not clear about the phrase *map drawn to a scale*.... A number of candidates made calculation errors. A few candidates made mistakes in unit conversion i.e., cm^2 to km^2.

(c) Several candidates used midpoint formula to find M instead of using section formula. Some candidates used section formula but made mistakes in substitution. Quite a few candidates found the values of x and y of M correctly but did not write the answer in the coordinate form. With the value of slope found being incorrect, equation of the required line was also incorrect.

Suggestions for teachers

- Stress upon the understanding of nature of roots and use of coefficient of x^2, x and constant, c for finding out the discriminant. Adequate practice could help in solving such problems.
- Give thorough practice in units and conversion of units to solve sums on maps and models. Also give adequate practice on size transformation and proportionality.
- Train students to express the points in coordinate geometry in coordinate form. Thorough drilling of section formula is necessary. Ensure that sufficient practice is given in finding slope of line and its equation using different methods.

MARKING SCHEME

Question 7

(a) Given equation is $x^2 + 4kx + (k^2 - k + 2) = 0$

For equal roots discriminant = 0

$\therefore (4k)^2 - 4(k^2 - k + 2) = 0$

$16k^2 - 4k^2 + 4k - 8 = 0$

$12k^2 + 4k - 8 = 0$

$3k^2 + k - 2 = 0 \Rightarrow 3k^2 + 3k - 2k = 0$

$3k(k + 1)(3k - 2) = 0$

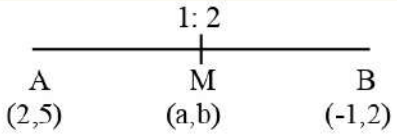
$\therefore k = -1, k = 2/3$

(b) AB = 6, BC = 8, Diagonal AC = $\sqrt{6^2 + 8^2} = 10$

(i) $\frac{10 \times 50000}{1000} = 5\ km$

(ii) $\frac{6 \times 8 \times 50000 \times 50000}{(100000)^2} = 12\ sq\ km$

(c)



$a = \frac{1 \times -1 + 2 \times 2}{1 + 2}$ $\quad b = \frac{1 \times 2 + 2 \times 5}{1 + 2}$

$a = \frac{-1 + 4}{3} = \frac{3}{3} = 1$ $\quad b = \frac{2 + 10}{3} = \frac{12}{3} = 4$

Co-ordinates of M (1,4)

Equation of MC:

M (1,4) C (5,8)

Slope = $\frac{8-4}{5-1} = \frac{4}{4} = 1$

$\therefore$ Equation is:

$y - 4 = 1\ (x - 1)$

$y - 4 = x - 1$

or $x - y + 3 = 0$

Question 8

(a) ₹7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received ₹100 more. Find the original number of children. [3]

(b) If the mean of the following distribution is 24, find the value of *'a'*. [3]

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of students	7	*a*	8	10	5

(c) Using ruler and compass only, construct a ΔABC such that BC = 5 cm and AB = 6.5 cm and ∠ABC = 120° [4]

(i) Construct a circum-circle of ΔABC

(ii) Construct a cyclic quadrilateral ABCD, such that D is equidistant from AB and BC.

Comments of Examiners

(a) Most candidates made mistakes in formulating the equation. Due to lack of concept on fractions some candidates formed the equation $\frac{7500}{x} - \frac{7500}{x-20} = 100$ instead of $\frac{7500}{x-20} - \frac{7500}{x} = 100$ Some were unable to factorize the equation correctly.

(b) Many candidates wrote incorrect Class mark of the given distribution. Hence, $\sum fx$ found was also incorrect. A few candidates wrote $\sum f$ as 30a instead of 30 + a and $\sum fx$ *was written as* $825a$ instead of 810 + 15a. Some made mistakes in applying the formula for mean and solving the equation to find 'a'.

(c) (i) Several candidates constructed triangle ABC with ∠ABC = 60° instead of 120°.

(ii) Quite a few candidates constructed circumcircle without bisecting two sides of the triangle so as to locate the circumcentre.

Suggestions for teachers

- In fractions, train students to differentiate which is greater or smaller.
- Train students to calculate mean involving unknown variables. Give them an understanding of finding class mark of grouped distribution. Revise basic concepts of algebraic operations.
- Give adequate practice of locus and geometrical properties. Advise students to show clearly all traces of construction using ruler and compass, while constructing geometrical figures.

(iii) Many candidates could not locate D, a point lying on the circle and bisector of ∠B. Hence could not complete the required quadrilateral.

MARKING SCHEME

Question 8

(a) Let the original number of children be x.

$\therefore$ each one gets $\frac{7500}{x}$

When 20 children less, each would get $\frac{7500}{x-20}$

$$\frac{7500}{x-20} - \frac{7500}{x} = 100$$

$$\Rightarrow x^2 - 20x - 1500 = 0$$

$$\Rightarrow (x-50)(x+30) = 0$$

$$\therefore x = 50$$

(b)

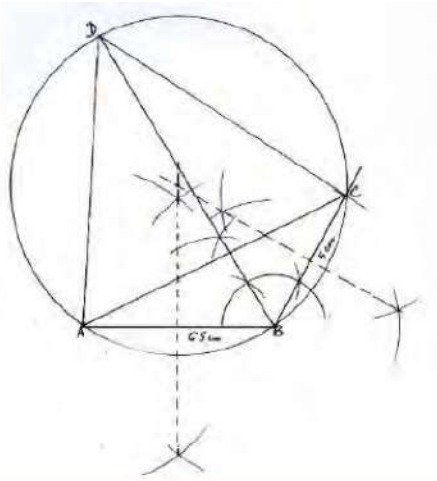
Marks	No. of Students(f)	x	fx
0 – 10	7	5	35
10 – 20	a	15	$15a$
20 – 30	8	25	200
30 – 40	10	35	350
40 – 50	5	45	225
	$\Sigma f = 30 + a$		$810 + 15a$

$$Mean = \frac{\Sigma fx}{\Sigma f}, \quad \frac{810+15a}{30+a} = 24$$

$$\therefore 810 + 15a = 720 + 24a$$

$$\therefore 810 - 720 = 24a - 15a$$

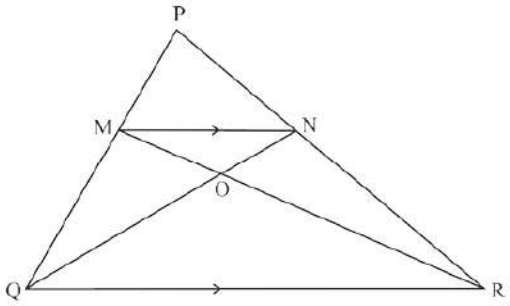
$$90 = 9a \therefore a = 10$$

(c)	Construction of ΔABC Construction of side bisectors of sides AB and BC and draw circle Angle bisector of angle B Cyclic quadrilateral ABCD	

Question 9

(a) Priyanka has a recurring deposit account of ₹1000 per month at 10% per annum. If she gets ₹5550 as interest at the time of maturity, find the total time for which the account was held. [3]

(b) In ΔPQR, MN is parrallel to QR and $\frac{PM}{MQ} = \frac{2}{3}$ [3]

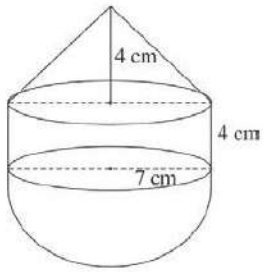


(i) Find $\frac{MN}{QR}$

(ii) Prove that ΔOMN and ΔORQ are similar.

(iii) Find, Area of ΔOMN : Area of ΔORQ

(c) The following figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find the volume of the solid. [4]



Comments of Examiners

(a) Many candidates considered ₹5550 as maturity value instead of interest. Some candidates got the correct equation $n^2 + n - 1332 = 0$, but were unable to factorize it correctly, hence, could not find 'n'.

(b) (i) A number of candidates were unable to prove Δ OMN ~ Δ ORQ

(ii) Δ OMN: Δ ORQ = 4:25 was written as $\frac{4}{25}$ hence many candidates went wrong with the answer.

(c) Many candidates did not read the question carefully and took the radius as $\frac{7}{2}$ instead of '7'. Some candidates made mistakes in applying the correct formula while a few made mistakes in calculation, especially those who used the value of $\pi = 3.14$

Suggestions for teachers

- Instruct students to read the question paper carefully to avoid making errors. e.g. taking ₹5550 as maturity value instead of interest. Stress upon revising middle term breakup factorization.
- More practice should be given to build up the concept of similarity and related results.
- Give sufficient practice in problems based on volume and surface area of different solids. Also teach students shorter methods of solving such problems instead of calculating each volume separately and then adding up.

MARKING SCHEME

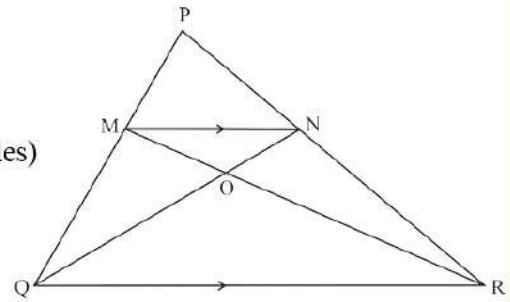
Question 9

(a) Monthly Deposit = ₹1000, rate of interest 10%

Total interest = ₹5550

Let n be the number of months

$$\therefore 5550 = \frac{1000 \times n(n+1) \times 10}{100 \times 2 \times 12}$$

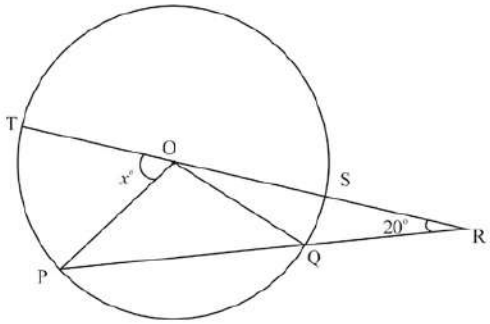
	$\frac{555 \times 24}{10} = n^2 + n \ \ or$ $n^2 + n - 1332 = 0$ $(n + 37)(n - 36) = 0$ $\therefore n = 36$ months i.e. $n = 3$ years
(b)	Proving $\Delta PMN \sim \Delta PQR$ (i) $\frac{MN}{QR} = \frac{2}{5}$ (ii) $\angle MON = \angle QOR$ (vertically opposite angles) $\angle MNO = \angle OQR$ (alternate angles) $\angle NMO = \angle ORQ$ (3rd angle) (iii) $ar\ \Delta OMN : ar\ \Delta ORQ = 4 : 25$ 
(c)	Volume = Volume of hemisphere + volume of cylinder + volume of cone $Volume = \frac{2}{3}\pi r^3 + \pi r^2 h + \frac{1}{3}\pi r^2 h$ $= \frac{\pi r^2}{3}[2r + 3h + h]$ $= \frac{22 \times 7^2}{7 \times 3}[2 \times 7 + 4 \times 4]$ $= \frac{22 \times 7}{3}[14 + 16]$ $= \frac{22 \times 7}{3} \times 30 = 1540\ cm^2$

Question 10

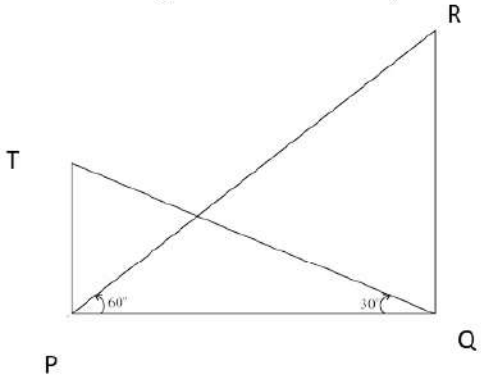
(a) Use Remainder theorem to factorize the following polynomial: [3]

$2x^3 + 3x^2 - 9x - 10.$

(b) In the figure given below 'O' is the centre of the circle. If QR = OP and ∠ORP = 20°. Find the value of 'x' giving reasons. [3]



(c) The angle of elevation from a point P of the top of a tower QR, 50 m high is 60° and that of the tower PT from a point Q is 30°. Find the height of the tower PT, correct to the nearest metre. [4]



Comments of Examiners

(a) Some candidates did not use Remainder theorem to identify the first factor as specified in the question. Some made mistakes in finding out the quotient by dividing the given polynomial by the factor found using factor theorem. Many candidates did not write the answer in the product form. e.g. $(x-2)(2x+5)(x+1)$

(b) A number of candidates could not identify the isosceles triangles OPQ and QOR. Some candidates could neither find the unknown angles, nor find the value of x. Reasons supporting the calculated result were either not given or given incorrect.

(c) Some candidates interchanged the values of tan 60° and tan 30° in the solution of the problem i.e., substituted the opposite of the correct values. Some made calculation errors. A few candidates did not round off the height of the tower to the nearest metre as per the requirement of the question.

Suggestions for teachers

- Instruct students to read the instruction given in the question carefully e.g., using remainder and factor theorem. Also guide them to express the final answer as the product of the factors obtained.
- Give more practice in properties of circles to enable students to solve problems based on circles. Importance of giving reasons to geometrical problems must be made clear to all students.
- More drilling is necessary in problems based on Heights and Distances.
- Instruct students to give the final answer in the correct form, as asked in the question.

MARKING SCHEME

Question 10

(a) $f(x) = 2x^3 + 3x^2 - 9x - 10$

$2x^3 + 3x^2 - 9x - 10$

$= 2 \times 2^3 + 3 \times 2^2 - 9 \times 2 - 10$ (Putting $x = 2$)

$= 16 + 12 - 10 - 10$

$= 0$ $\therefore (x-2)$ is a factor

$2x^3 + 3x^2 - 9x - 10 = (x-2)(2x^2 + 7x + 5)$

$= (x-2)(2x+5)(x-2)$

(b)

$\because OP = OQ \quad (radii\ of\ circle)$

$and\ OP = QR \quad (given)$

$\therefore OP = OQ = QR$

$In\ \Delta OQR, \quad OQ = QR$

$\therefore \angle ORQ = \angle ROQ = 20°$

$ext.\ \angle OQP = 20° + 20° = 40°$ (exterior angle = sum of 2 interior opposite angles)

$In\ \Delta OPQ, OP = OQ$ (radii)

$\therefore \angle OPQ = \angle OQP = 40°$

$In\ \Delta OPR, ext\ \angle POT = \angle OPR + \angle ORP = 40° + 20° = 60°$

(c)

In ΔPQR,

$$tan60° = \frac{QR}{PQ} = \frac{50}{PQ} \Rightarrow PQ = \frac{50}{\sqrt{3}}$$

In ΔPQT, OR

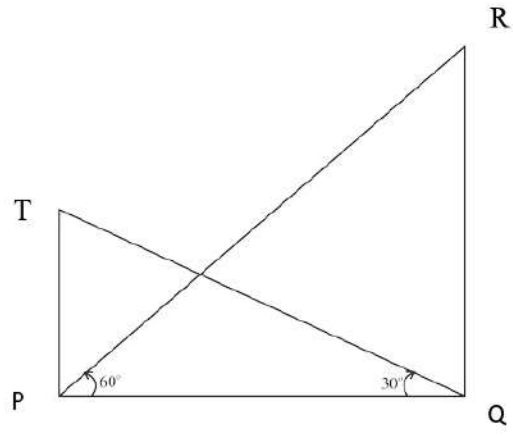
$$tan30° = \frac{PT}{PQ} \Rightarrow \frac{1}{\sqrt{3}} = \frac{PT}{50/\sqrt{3}}$$

$$\Rightarrow PT = \frac{50}{3} = 16.66$$

= 17 m (rounded to nearest metre)

$tan60 = \sqrt{3}\ \ OR\ \ tan30 = \frac{1}{\sqrt{3}}$

PT = 17 m



Question 11

(a) The 4^{th} term of an A.P. is 22 and 15^{th} term is 66. Find the first term and the common difference. Hence find the sum of the series to 8 terms. [4]

(b) Use Graph paper for this question. [6]

A survey regarding height (in cm) of 60 boys belonging to Class 10 of a school was conducted. The following data was recorded:

Height in cm	135 – 140	140 – 145	145 – 150	150 – 155	155 – 160	160 – 165	165 – 170
No. of boys	4	8	20	14	7	6	1

Taking 2 cm = height of 10 cm along one axis and 2 cm = 10 boys along the other axis draw an ogive of the above distribution. Use the graph to estimate the following:

(i) the median

(ii) lower Quartile

(iii) if above 158 cm is considered as the tall boys of the class. Find the number of boys in the class who are tall.

Comments of Examiners

(a) A number of candidates were unable to form the equations with given conditions. Some candidates, without using given conditions, guessed the series and identified the values of 'a' and 'd'. There were a few candidates who did not know the formula for summation, who simply surmised the series up to 8 terms and added them to find the sum.

(b) Following errors were observed in this question:
- The last cumulative frequency did not tally with the total of the given distribution.
- Some candidates chose incorrect scale.
- Perpendicular lines were not dropped to find the values from the ogive.
- The ogive was plotted with respect to lower boundaries instead of upper boundaries.
- Ruler was used to draw the graph instead of a freehand curve.
- Kink was not shown on the graph sheet.

Suggestions for teachers

- Explain arithmetic progression and geometric progression comprehensively to students and give ample practice on simplification of problems based on both types of series.
- Explain the concept of kink clearly to students. Also give more practice in ogive using graph sheet.

MARKING SCHEME

Question 11

(a) Let the first term be *'a'* and the common difference *d*.

$T_n = a + (n-1)\,d$

$\therefore T_4 = a + (4-1)\,d, \quad T_{15} = a + (15-1)\,d$

$22 = a + 3d \qquad 66 = a + 14d$

$a + 14d = 66$ and $a + 3d = 22$

Solving $a = 10$, $d = 4$

$S_n = \frac{n}{2}[2a + (n-1)\,d$

$S_8 = \frac{8}{2}[2 \times 10 + (8-1)\,4$

$= 4[20 + 28]$

$\therefore S_8 = 192$

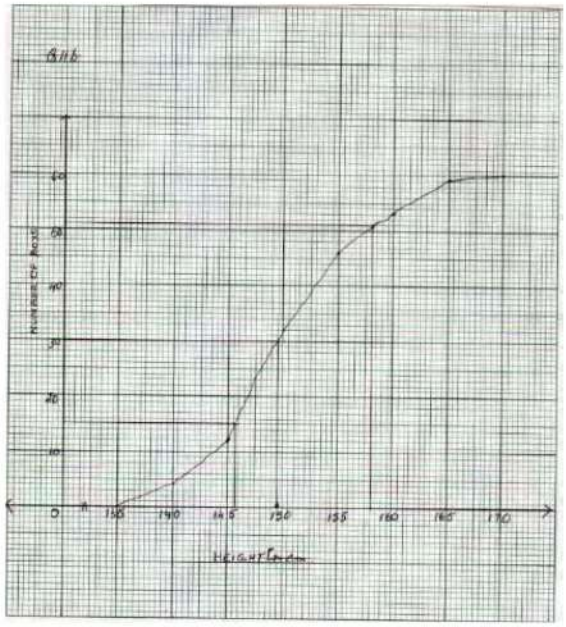
(b)

Height n cm	No. of boys	*cf*
135 – 140	4	4
140 – 145	8	12
145 – 150	20	32
150 – 155	14	46
155 – 160	7	53
160 – 165	6	59
165 – 170	1	60

(i) Median = 150 (±1)

(ii) Q_1 = 146 (±1)

(iii) 60 – 51 = 9 boys



Note: For questions having more than one correct answer/solution, alternate correct answers/solutions, apart from those given in the marking scheme, have also been accepted.

GENERAL COMMENTS

Topics found difficult/ confusing by candidates

- Arithmetic Progression and Geometric Progression.
- Shares and Dividend.
- Geometry based problems using properties of circle and similar triangles.
- Geometrical Constructions.
- Section formula.
- Trigonometrical Identities and Heights and Distances.
- Properties of Ratio and Proportion.
- Approximation: to given significant figures or to nearest whole number.
- Factorization in various problems.
- Maps and models.
- Maturity value and interest in Banking sums.
- Use of Remainder and Factor Theorem.
- Forming correct equation in Quadratic word problem.
- Inequation solving and representing solution on number line.

Suggestions for candidates

- Utilize the reading time to make the right choice of question and to be thorough with the given data.
- Practise approximation like significant figures, rounding off to a certain place of decimal or nearest whole number extensively.
- Practise plotting graphs using the correct scale, reading values/interpretation from graph.
- Clearly show the traces of geometrical constructions.
- Give reasons while working out Geometry based problems.
- Practise trigonometric basic identities comprehensively.
- Practise the concepts of Coordinate Geometry- section formula, slope of a parallel/perpendicular line, equation of a line.
- Concepts of Arithmetic Progression(AP) and Geometric Progression(GP) must be done meticulously.
- Use Mathematical tables to find square roots and ratios other than that of standard angles.
- All steps of working including rough work must be clearly shown on same answer page.
- Study the entire syllabus completely and revise from time to time.
- Revise the concepts learnt in Class IX and integrated with the Class X syllabus.
- Develop logical and reasoning skills to have a clear understanding of the concepts.

ICSE Class 10 Maths Question Paper Solution 2019

SECTION A (40 Marks)

*Attempt **all** questions from this Section.*

Question 1

(a) Solve the following inequation and write down the solution set: [3]

$$11x - 4 < 15x + 4 \leq 13x + 14, x \epsilon W$$

Represent the solution on a real number line.

(b) A man invests ₹4500 in shares of a company which is paying 7.5% dividend. [3]

If ₹100 shares are available at a discount of 10%.

Find:

(i) Number of shares he purchases.

(ii) His annual income.

(c) In a class of 40 students, marks obtained by the students in a class test (out of 10) are given below: [4]

Marks	1	2	3	4	5	6	7	8	9	10
Number of students	1	2	3	3	6	10	5	4	3	3

Calculate the following for the given distribution:

(i) Median

(ii) Mode

Comments of Examiners

(a) Many candidates made errors in transposing like terms on same side e.g., $15x - 11x <- 4 - 4$. Some candidates made mistakes in solving the inequation. Most of the errors were pertaining to positive and negative signs like, $-4x < 8 \Rightarrow x < -2$

A number of candidates did not write down the solution set after solving the given inequation. Some did not use set notation method with curly brackets, and some represented as real number solution stating $x \in R$. Several candidates represented solution incorrectly on the number line i.e., $X \in N$ was considered as $x \in R$. A few candidates failed to put extra number on each side of solution for indicating the continuity of the number line.

(b) (i) Common errors made by many candidates were in finding annual income, in considering Market Value (MV) i.e., instead of taking discounted MV as ₹90 it was taken as either ₹100 or ₹110. Thus, made mistake in finding the number of shares purchased.

(ii) Annual income was also calculated incorrectly due to error in finding the number of shares.

(c) (i) A large number of the candidates were unable to understand that the given data is a non-grouped frequency distribution. To identify the median, instead of finding the cumulative frequency, they tried to find it directly. Some tried to find the median by plotting the points directly on a graph paper. Some candidates tried to find the median by writing all 40 numbers.

(ii) Several candidates could not identify mode correctly.

Suggestions for teachers

- Revise the concepts of number system viz., Natural numbers (N), Whole numbers (W), Integers (Z), Real numbers (R) frequently.
- Give adequate practice for transpositions of variables and constants and division by negative number in inequations.
- Teach the method of writing the solution set in set notation form.
- Clarify the rules of representation of the solution on the number line whether it belongs to N, W, Z or R.
- Build the concept of Nominal Value (NV), Market Value (MV), dividend etc by giving examples.
- Discuss the Simple distribution, Non-grouped frequency distribution and Grouped frequency distribution in detail with corresponding method of measure of their central tendencies.
- Drill students with basic concepts like inequations, shares & dividends, median, mode and related application-based problems.

MARKING SCHEME

Question 1

(a) $11x - 4 < 15x + 4 \leq 13x + 14, x \epsilon W$

$11x - 4 < 15x + 4$	$15x + 4 \leq 13x + 14$	(Transforming x terms on one side and constants on the other side)
$11x - 15x < 4 + 4$	$15x - 13x \leq 14 - 4$	
$-4x < 8$	$2x \leq 10$	
$+4x > -8$	$x \leq 5$	
$x > -2$		

Solution: {0, 1, 2, 3, 4, 5}

(b) (i) Market value of each share = ₹100 – 10% of ₹100 = ₹90

$\therefore$ Number of shares = $\frac{4500}{90} = 50$

(ii) Annual Income = $\frac{7.5 \times 100 \times 50}{100}$ = ₹375

(c)

Marks (x)	No. of Students (f)	cf
1	1	1
2	2	3
3	3	6
4	3	9
5	6	15
6	10	25
7	5	30
8	4	34
9	3	37
10	3	40
	$\Sigma = 40$	

(i) Median = 6

(ii) Mode = 6

Question 2

(a) Using the factor theorem, show that $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$. [3]

Hence factorise the polynomial completely.

(b) Prove that: [3]

$$(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$$

(c) In an Arithmetic Progression (A.P.) the fourth and sixth terms are 8 and 14 respectively. [4]

Find the:

(i) first term

(ii) common difference

(iii) sum of the first 20 terms.

Comments of Examiners

(a) A large number of candidates did not use Remainder-Factor Theorem to show that (x – 2) is a factor of the given polynomial. Few of them did not conclude the remainder to be equal to 0. In some scripts, errors were observed in division of the polynomial by (x – 2). Many candidates expressed the final answer by separating the factors by comma instead of expressing them in the product form as (x – 2) (x+1) (x+2).

(b) Many candidates made mistakes in substituting the reciprocal relations for the trigonometric ratios given in the question for example, cosec θ is taken as 1/ cos θ instead of 1/sin θ etc. while some candidates made mistakes in simplification of the expressions on the Left-Hand Side of the question. Some candidates made mistakes in using the identities like $\sin^2 \theta + \cos^2 \theta = 1$ correctly. Some expanded the whole expression and made mistakes in simplification and using standard identities.

(c) Some candidates did not know the basic concepts of Arithmetic Progression (A.P.). Many candidates tried to solve the sum by trial & error method and got the six terms, but the necessary working was incorrect. Few candidates without framing equations like a+3d = 8 and a+5d = 14, wrote d=3. Some candidates either used incorrect formulae or made errors in calculation. Therefore, they failed to get the value of the first term, common difference and sum of the first 20 terms correctly.

Suggestions for teachers

- Clarify the factor theorem / Remainder theorem thoroughly with examples.
- Advise students to write the factors in product form.
- Drilling of simple trigonometric identities to apply them appropriately in solving other identities.
- Insist upon showing working clearly.
- Teach the concept of the two series: Arithmetic Progression (AP) and Geometric Progression (GP) and their differences with different examples.
- Revise formulae for finding a term, common difference and summation of certain number of terms in Arithmetic Progression frequently.

MARKING SCHEME

Question 2

(a)

$f(x) = x^3 + x^2 - 4x - 4$

$f(2) = 2^3 + 2^2 - 4 \times 2 - 4$

$= 8 + 4 - 8 - 4$

$= 0$

$\therefore\ x - 2$ is a factor of $f(x)$

$x - 2 \mid x^3 + x^2 - 4x - 4 \mid x^2 + 3x + 2$

$$\begin{array}{r} x^3 - 2x^2 \\ \hline 3x^2 - 4x \\ 3x^2 - 6x \\ \hline 2x - 4 \\ 2x - 4 \\ \hline x \end{array}$$

$x^2 + 3x + 2 = (x + 1)(x + 2)$

$\therefore x^3 + x^2 - 4x - 4 = (x - 2)(x + 1)(x + 2)$

(b)

$(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

$$LHS - \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1 - \sin^2\theta}{\sin\theta}\right) \times \left(\frac{1 - \cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$$

$$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \frac{1}{\sin\theta\cos\theta}$$

$= 1 = RHS$

(c) Let a be the first term and d the common difference

$\therefore a + 3d = 8$ and $a + 5d = 14$

(i) Solving $a = -1$

(ii) $d = 3$

(iii) $S_n = \frac{n}{2}\{2 \times a + (n-1)\,d\}$

$S_{20} = \frac{20}{2}\{2 \times (-1) + (20-1)\,3\}$

$S_{20} = 10(-2 + 57)$

$S_{20} = 550$

Question 3

(a) Simplify [3]

$$\sin A \begin{bmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{bmatrix} + \cos A \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$$

(b) M and N are two points on the X axis and Y axis respectively. [3]

P (3, 2) divides the line segment MN in the ratio 2: 3.

Find:

(i) the coordinates of M and N

(ii) slope of the line MN.

(c) A solid metallic sphere of radius 6 cm is melted and made into a solid cylinder of height 32 cm. Find the: [4]

(i) radius of the cylinder

(ii) curved surface area of the cylinder

Take $\pi = 3.1$

Comments of Examiners

(a) Some candidates made mistakes in scalar multiplication of the matrix. For Example,

$$\sin A \begin{bmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{bmatrix} = \begin{bmatrix} sin^2 A & \sin A - \cos A \\ \sin A \cos A & sin^2 A \end{bmatrix}$$

Some made errors in matrix addition and did not apply the identity $sin^2A+cos^2A = 1$ for simplifying the final matrix. A number of candidates put some specific value, e.g., $90°, 60°$ *etc* and tried to solve the problem.

(b)(i) Several candidates made errors in identifying coordinates of points on x-axis and y-axis, e,g., (x,0) and (0,y) was taken as (0,x) , (y,0) or as (x_1, y_1) and (x_2, y_2).
Many candidates instead of using the given ratio $MP: PN = 2: 3$ took $NP: MP = 2: 3$.
Some candidates used midpoint formula to find coordinates of P. A number of candidates made calculation errors. Some overlooked the last part of the question and did not find slope of the line MN.

(ii) As some candidates obtained incorrect coordinates of M and N, they made error in finding the slope of the line MN. Some candidates were not conversant with the formula to find the slope of a line.

(c) (i) Some candidates used incorrect formula for finding the volume of a sphere and a cylinder, e.g., volume of sphere was taken as $4/3\ \pi r^2$ or $2/3\ \pi r^3$ and volume of cylinder was taken as $2\pi r^2 h$ or $\pi \times 6^2 \times 32$ instead of $\pi r^2\ 32$.Some candidates did not use the value of π given in the question paper, hence, got incorrect answer.

(ii) Many candidates used incorrect formula for finding curved surface area of the cylinder, e.g., used $2\pi r^2\ h$ instead of $2\pi rh$.

Suggestions for teachers

- Give thorough practice of basic operations like addition, multiplication of matrices and solving matrix equation.
- Instruct students to show each step of working and also to use matrix notation for each step.
- Advise students to read the question carefully and take note of every part to be attempted.
- Familiarise students with the logic of assigning coordinates to the points taken on the x-axis and on the y-axis.
- Teach students that if a point divides a line segment in the ratio m:n ($m \neq n$) then m: n is not equal to n: m.
- Give adequate practice to the students for application of various mensuration formulae of volume and surface area of solids.
- Give revision of volume and surface area related application-based problems to the students to clarify their concepts.
- Train students to handle carefully the operation of multiplication and division with decimal numbers.

MARKING SCHEME

Question 3

(a)

$$\sin A \begin{bmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{bmatrix} + \cos A \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2 A & -\sin A \cos A \\ \sin A \cos A & \sin^2 A \end{bmatrix} + \begin{bmatrix} \cos^2 A & \cos A \sin A \\ -\sin A \cos A & \cos^2 A \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2 A + \cos^2 A & -\sin A \cos A + \cos A \sin A \\ \sin A \cos A - \sin A \cos A & \sin^2 A + \cos^2 A \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(b) Let M(x, 0) and N(0,0) and N(0, y) be the two points on the X and Y axis respectively.

(i) P(3, 2) divides MN in the ratio 2 : 3

$\therefore\ 3 = \frac{2\times0+3\times x}{2+3}$ $\quad\therefore x = 5$

$2 = \frac{2\times y+3\times0}{2+3}$ $\quad\therefore y = 5$

M (5, 0), N(0, 5)

(ii) Slope of MN: $\frac{5-0}{0-5} = -1$

(c) Let the radius of the cylinder be 'r'

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\pi\times6^3$

Volume of cylinder $= \pi r^2 h = \pi\times r^2\times32$

(i) $\therefore \frac{4}{3}\pi\times6^3 = \pi\times r^2\times32$

$\therefore r^2 = \frac{4\times6^3}{3\times32} = 3^2$

$\therefore r = 3\ cm$

(ii) Curved surface area of cylinder $= 2\pi rh$

$= 2\times3.1\times3\times32$

$= 595.2\ cm^2$

Question 4

(a) The following numbers, K + 3, K + 2, 3K – 7 and 2K – 3 are in proportion. Find K. [3]

(b) Solve for x the quadratic equation $x^2 - 4x - 8 = 0$. [3]

Give your answer correct to three significant figures.

(c) Use ruler and compass only for answering this question. [4]

Draw a circle of radius 4 cm. Mark the centre as O. Mark a point P outside the circle at a distance of 7 cm from the centre. Construct two tangents to the circle from the external point P.

Measure and write down the length of any one tangent.

Comments of Examiners

(a) Some candidates did not use the property of proportion correctly. Many candidates used arithmetic progression to solve the sum instead of ratio and proportion. A number of candidates used componendo and dividendo but their working was incorrect. Several candidates did not equate the expression $k^2 - 4k - 5$ to zero. A few candidates neglected the final answer 5 and -1.

(b) Many candidates used incorrect formula for finding roots of the quadratic equation. Some used correct formula but incorrect substitution, e.g., c = 8 instead of – 8. Many candidates could not express $\sqrt{48}$ as $4\sqrt{3}$ and a few failed to substitute $\sqrt{3}$=1.732. Some candidates used long division method to find $\sqrt{48}$ and went wrong in calculation. Several candidates did not express the final answer up to three significant figures as it was asked in the question.

(c) Several candidates did not take distance OP = 7cm as it was given in the question. Many candidates marked midpoint of OP using ruler instead of constructing the perpendicular bisector. Some candidates followed some other incorrect methods for construction. A number of candidates measured the length of the tangent incorrectly.
Some candidates wasted time in writing the steps of construction which was not asked in the question.

Suggestions for teachers

- Advise students to read the question carefully and apply the condition correctly as given in the problem.
- Train students to analyse a given question with what is given and what is to be evaluated.
- Acquaint students with the use of mathematical tables to get the value of $\sqrt{48}$ directly instead of wasting time to get the value using division method which is not necessary.
- Clarify significant figures thoroughly with relevant examples.
- Insist upon giving all traces of construction.
- Instruct students to read the construction-based question carefully and solve the problem as per its requirement.

MARKING SCHEME

Question 4

(a) $K+3, K+2, 3K-7$ and $2K-3$ are in proportion

$$\therefore \frac{K+3}{K+2} = \frac{3K-7}{2K-3}$$

Or $(K+3)(2K-3) = (3K-7)(K+2)$

$$2K^2 + 6K - 3K - 9 = 3K^2 - 7K + 6K - 14$$

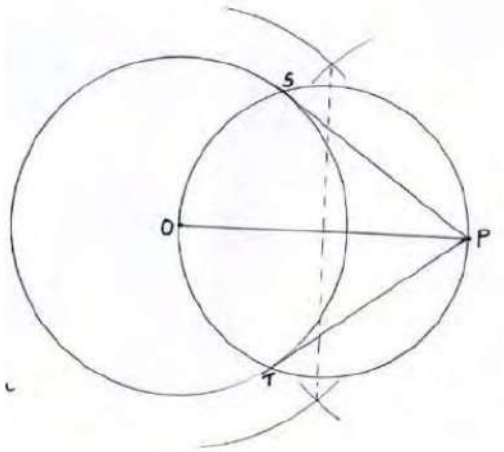
$$K^2 - 4K - 5 = 0$$

$$(K-5)(K+1) = 2$$

$$\therefore K = 5, -1$$

(b) $x^2 - 4x - 8 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4.1.(-8)}}{2 \times 1}$$

	$= \frac{4 \pm \sqrt{16+32}}{2}$ $= \frac{4 \pm \sqrt{48}}{2}$ $= \frac{4 \pm 4\sqrt{3}}{2}$ $= 2 \pm 2\sqrt{3} = 2 \pm 2 \times 1.732$ $x = 2 \pm 3.464$ $\therefore x = 5.464 \text{ or } -1.464$ $x = 5.46, -1.46$ (*Correct to three significant figures.*)
(c)	Circle and point P Bisecting OP and drawing circle Two tangents  PS = PT = (5.7 ± 0.2) cm

SECTION B (40 Marks)

*Attempt any **four** questions from this Section*

Question 5

(a) There are 25 discs numbered 1 to 25. They are put in a closed box and shaken thoroughly. A disc is drawn at random from the box. [3]

Find the probability that the number on the disc is:

(i) an odd number

(ii) divisible by 2 and 3 both.

(iii) a number less than 16.

(b) Rekha opened a recurring deposit account for 20 months. The rate of interest is 9% per annum and Rekha receives ₹441 as interest at the time of maturity. [3]

Find the amount Rekha deposited each month.

(c) Use a graph sheet for this question. [4]

Take 1 cm = 1 unit along both x and y axis.

(i) Plot the following points:

A(0,5), B(3,0), C(1,0) and D(1,–5)

(ii) Reflect the points B, C and D on the y axis and name them as B', C' and D' respectively.

(iii) Write down the coordinates of B', C' and D'.

(iv) Join the points A, B, C, D, D', C', B', A in order and give a name to the closed figure ABCDD'C'B'.

Comments of Examiners

(a) (i) Common errors noticed in sub parts (i) ,(ii) and (iii) of this question were:

- total outcome or favourable outcome was incorrect in all parts.
- favourable outcome of odd numbers was taken as 13.
- numbers divisible by 2 and 3 both were listed incorrectly. Many candidates misread the condition as 2 or 3. Hence, favorable outcomes was 16 instead of 4.
- the probability was not expressed in the simplest form, e.g., 15/25 was not written as 3/5.

(b) Some candidates considered ₹441 as maturity value and some considered it as monthly installment instead of taking it as interest.
Many candidates either used incorrect formula or made errors in calculation.

(c) Following anomalies were observed in the solution of graph-based question:

- incorrect choice of scale i.e., 2cm = 1unit instead of 1cm = 1unit as it was given in the question paper.
- marked axes incorrectly.
- plotted A (0,5), B (3,0) and C(1,0) incorrectly.
- plotted the points correctly but did not label them.
- neither join the points nor completed the figure.
- reflected points B', C', and D' were marked incorrectly.
- coordinates of B', C', and D' were not written correctly.
- did not name the closed figure as given in the question.

Suggestions for teachers

- Instruct students to list the outcomes of both total and favourable events.
- Concept of AND, OR in finding the probability of a number on the disc divisible by 2 and 3 or 2 or 3 require more drilling.
- In probability, emphasise in giving answers in the simplest form.
- Instruct students to read the question carefully so as not to go wrong with given data.
- More drilling of calculation based sums minimise calculation errors.
- Formulae for finding the monthly installment, MV or number of months need repeated drilling.
- Candidates must be instructed to read each part carefully for a question on reflection.
- More practice required on problems leading to Graphs with proper choice of axis.
- Give adequate practice to the students to identify points on the x and y axis in the class.

MARKING SCHEME

Question 5

(a) Total number of outcomes = 25

(i) favourable outcomes are {1, 3, 5, 7, 11, 13, 15, 17, 19, 21, 23, 25}

$\therefore$ probability of being an odd number is $= \frac{12}{25}$

(ii) favourable outcomes are {6, 12, 18, 24} i.e. 4

$\therefore$ probability of being divisible by both 2 and 3 is $= \frac{4}{25}$

(iii) favourable outcomes are
{1, 2, 3, 4, 5…15}, i.e. 15

$\therefore$ probability $= \frac{15}{25} = \frac{3}{5}$

(b) Let monthly deposit be ` x

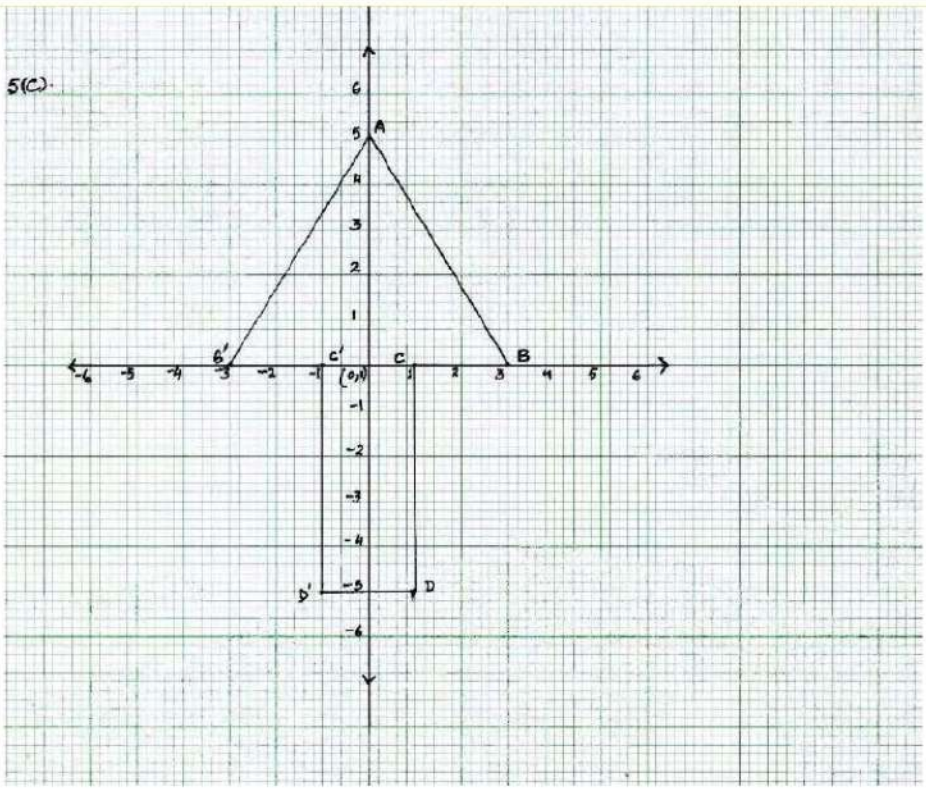
Interest $= \frac{P \times n(n+1) \times r}{100 \times 2 \times 12}$

$$441 = \frac{x \times 20(20+1) \times 9}{100 \times 2 \times 12}$$

$\frac{63x}{40} = 441$ Equating interest to 441

$\therefore x = ₹280$

(c)



(iii) B(3, 0) $\xrightarrow{y-axis}$ B'(–3, 0)

C(1, 0) $\xrightarrow{y-axis}$ C'(–1, 0)

D(1, –5) $\xrightarrow{y-axis}$ D'(–1, –5)

Plotting A, B, C, D

Reflected points B',C',D'

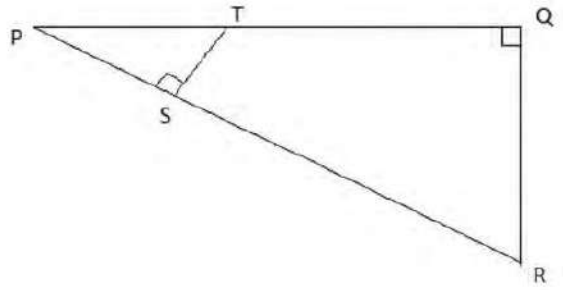
(iv) Arrow Head / Heptagon(or Septagon)

Question 6

(a) In the given figure, $\angle PQR = \angle PST = 90°$, PQ = 5 cm and PS = 2 cm. [3]

(i) Prove that $\Delta PQR \sim \Delta PST$.

(ii) Find Area of ΔPQR: Area of quadrilateral SRQT.



(b) The first and last term of a Geometrical Progression (G.P.) are 3 and 96 respectively. If the common ratio is 2, find: [3]

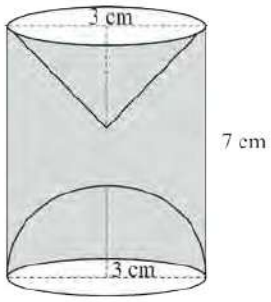
(i) '***n***' the number of terms of the G.P.

(ii) Sum of the **n** terms.

(c) A hemispherical and a conical hole is scooped out of a solid wooden cylinder. Find the volume of the remaining solid where the measurements are as follows: [4]

The height of the solid cylinder is 7 cm, radius of each of hemisphere, cone and cylinder is 3 cm. Height of cone is 3 cm.

Give your answer correct to the nearest whole number. Take $\pi = \frac{22}{7}$.



Comments of Examiners

(a) (i) Many candidates could not identify the two sets of equal angles to prove $\Delta PQR \sim \Delta PST$

(ii) Several candidates made mistake in writing (ΔPQR)/ΔPST=25/4. Hence, could not find area of ΔPQR: area of quadrilateral $\square SRQT$.
Some candidates left the answer as 25/21 instead of writing 25:21 as mentioned in the Question.

(b) (i) A number of candidates were unable to form the equations with given conditions. Due to cconceptual errors related to Geometrical Progression many candidates were unable to find the value of n. Some tried to find n by trial and error method.

(ii) Many candidates could not find the value of n but found the sum by using S=(lr-a)/(r-1) and got the correct answer.
Some candidates used incorrect formula for finding the sum of n terms.

(c) Some candidates made mistakes in applying the correct formula of one or more of solids (cylinder, hemisphere and cone) while a few made mistakes in substitution of values of radius and height of the solid/s scooped out of the cylinder. Some candidates took the value of $\pi = 3.14$ for calculation instead of the given value 22/7. A few candidates made mistakes in rounding of the answer to the nearest whole number. Basic calculation errors were also noticed in many answers scripts.

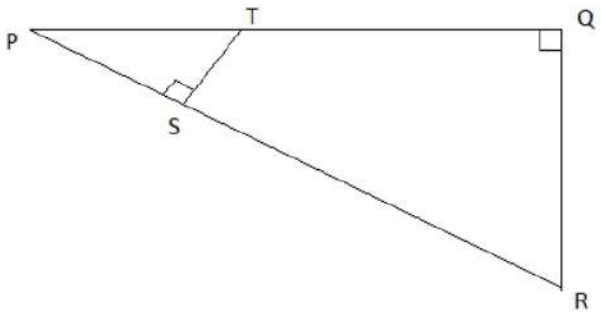
Suggestions for teachers

- Build up the concept of similarity of triangles and related results by giving adequate practice and frequent class tests.
- Advise students to write down the proportional sides after proving similarity.
- Instruct students to read the question carefully and underline the points to be answered and the correct form of the answer.
- Advise students to understand the formulae related to sequence and series with reference to the context and use them to solve the problems by following the correct method and clearly showing all steps of working.
- Train students to calculate the volume of a remaining solid by putting all the volumes in a combine manner with plus and minus sign instead of calculating each volume separately.
- Give sufficient practice in problems based on volume and surface area of different solids.
- Read the question carefully and use the given value of π. Use $\pi = 22/7$ in such sums unless otherwise if it is mentioned.
- More drilling of such sums could rectify the calculation errors.

MARKING SCHEME

Question 6

(a)



(i) In ΔPQR and ΔPST

$\angle PQR = \angle PST = 90^\circ$ (given)

$\angle P$ is common to both triangles.

$\therefore \Delta PQR \sim \Delta PST$ **(AAA)**

(ii) $\therefore \Delta PQR : \Delta PST = 5^2 : 2^2$

$\therefore \Delta PQR$: quadrilateral SRQT = 25: 21

(b) 1st term $a = 3$ and last term = 96, $r = 2$

$\therefore T_n = 96 = a\, r^{n-1}$

$96 = 3 \times 2^{n-1}$

$32 = 2^{n-1}$

$2^5 = 2^{n-1}$

$\therefore n-1 = 5$ hence $n = 6$

$$S_6 = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{3(2^6 - 1)}{2 - 1} = 3(64 - 1) = 3 \times 63 = 189$$

(c) Remaining volume = Volume of Cylinder – (volume of cone + volume of hemisphere)

$$= \pi r^2 h - \left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)$$

$$= \frac{22}{7} \times 3^2 \times 7 - \left(\frac{1}{3} \times \frac{22}{7} \times 3^2 \times 3 + \frac{2}{3} \times \frac{22}{7} \times 3^3\right)$$

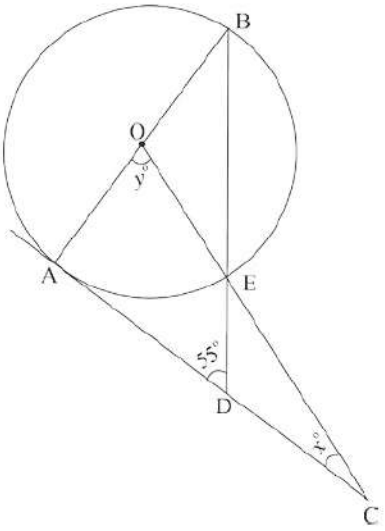
$$= \frac{22}{7} \times 3^2[7 - (1 + 2)]$$

$$= \frac{22}{7} \times 3^2 \times 4 = \frac{792}{7} = 113.1 = 113\ cm^3$$

Question 7

(a) In the given figure AC is a tangent to the circle with centre O. [3]

If ∠ADB = 55°, find x and y. Give reasons for your answers.



(b) The model of a building is constructed with the scale factor 1 : 30. [3]

(i) If the height of the model is 80 cm, find the actual height of the building in meters.

(ii) If the actual volume of a tank at the top of the building is 27 m^3, find the volume of the tank on the top of the model.

(c) Given $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = 6\,I$, where M is a matrix and I is unit matrix of order 2x2. [4]

(i) State the order of matrix M.

(ii) Find the matrix M.

Comments of Examiners

(a) Some candidates were unable to identify $\angle BAD = 90°$ in the given figure. Hence, could not find $\angle ABD$, thereby got incorrect answers of x and y.
A number of candidates considered OADE as a cyclic quadrilateral and hence went wrong with the sum.
Many candidates solved the question without giving proper reasoning.
In some scripts simple calculation errors were also observed.

(b) Many candidates were not clear about the *scale factor*. A number of candidates made calculation errors. A few candidates made mistakes in unit conversion i.e., cm to m and cm^3 to m^3 for example took 2400 cm = 240 m and 100 cm^3 = 0.001 m^3.
A few of them left the answer in proper fraction as 1/1000 without any unit and some left answer to subpart (ii) as 1000 without any unit.

(c) (i) Some candidates were unable to find the order of the matrix.
Many were not aware that the matrix multiplication is not commutative and changed matrix M from right side in the given question to left side

$$i.e., \quad \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = 6I \text{ to } M \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} = 6I$$

Some took the identity matrix as $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ instead of $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(ii) Several candidates could not find the matrix multiplication correctly. Some candidates made mistake in framing the simultaneous equations and some could not find the values of the unknowns. A few got the values of the unknowns but did not arrange them in a matrix form.

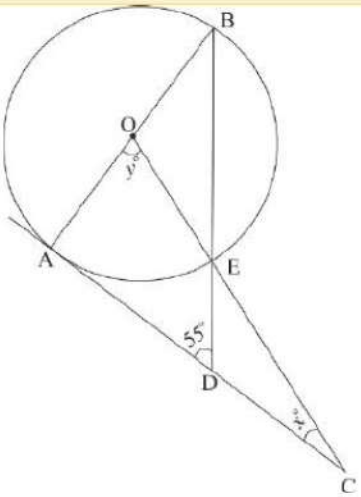
Suggestions for teachers

- Exhaustive drill of properties of circle theorems and on application-based sums is must.
- Train students to give reasons supporting the answers while solving geometry-based questions.
- Teach students to name the angles with three letters, specifically when there are two or more angles at the same point, e.g., ∠OED must not be named as ∠E as it may mean ∠ CED or ∠OEB or ∠BEC.
- Give sufficient practice of application-based problems on size transformation and proportionality.
- Intensive drill in conversion of units is must.
- Explain the concept of area, proportional to square of sides and concept of volume proportional to cube of sides.
- Instruct students to show each step of matrix multiplication.
- Ensure that enough practice is given in solving simultaneous equations.
- Train students to solve the sums involving identity matrix.

MARKING SCHEME

Question 7

(a)



AC is a tangent and OA is the radius.

$\therefore \angle BAC = 90^\circ$

In ΔABD

$\angle BAD = 90^\circ$

$\therefore \angle B = 180^\circ - (90^\circ + 55^\circ)$

$= 35^\circ$ (angles of a triangle adds upto 180°)

$\angle y = 2\angle B$

$\therefore y = 2 \times 35^\circ = 70^\circ$ (angle at the centre is double the angle in the remaining circumference)

In ΔAOC

$x = 180^\circ - (90^\circ + y)$

$= 180^\circ - (90^\circ + 70^\circ)$

$= 20^\circ$ (angles of a triangle adds upto 180°)

(b) $K = \frac{1}{30}$ Used correctly in height or volume

(i) Height of model $= \frac{1}{30} \times$ Actual height of the building

$= 30 \times 80$ cm = Actual height of the building

Actual height of the building = 2400 cm = 24 m

OR

1: 30 = 80 : x (where x is the actual height of the building)

$\therefore x = 2400$ cm = 24 m.

	(ii)	Volume of the tank on the top of the model $= \left(\frac{1}{k}\right)^3 \times$ Actual volume of tank at the top of the building $= \frac{1\times27}{30\times30\times30}\ m^3$ $= \frac{27\times100\times100\times100}{30\times30\times30}\ cm^3$ $= 1000\ cm^3$
(c)	(i)	$(2\times 2)(m\times n) = (2\times 2) \rightarrow$ order of matrix, M = (2×2)
	(ii)	$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 4a+2c & 4b+2d \\ -a+c & -b+d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$ $4a + 2c = 6$ $-a + c = 0$ $\therefore a = 1\ and\ c = 1$ $4b + 2d = 0$ $-b + d = 6$ $\therefore b = -2\ and\ d = 4$ $\therefore M = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$

Question 8

(a) The sum of the first three terms of an Arithmetic Progression (A.P.) is 42 and the product of the first and third term is 52. Find the first term and the common difference. [3]

(b) The vertices of a ΔABC are A(3, 8), B(–1, 2) and C(6, –6). Find: [3]

(i) Slope of BC.

(ii) Equation of a line perpendicular to BC and passing through A.

(c) Using ruler and a compass only construct a semi-circle with diameter BC = 7cm. Locate a point A on the circumference of the semicircle such that A is equidistant from B and C. Complete the cyclic quadrilateral ABCD, such that D is equidistant from AB and BC. Measure ∠ADC and write it down. [4]

Comments of Examiners

(a) Many candidates took a-d, a and a+d as three terms of Arithmetic Progression and found a=14 but while answering considered a as the first term instead of a-d. A number of candidates did not write d=±12 but took d= +12 and hence found only one set of values.
Some candidates could not form the correct equation with the given data.

(b) Many candidates found the slope of the line BC correctly but failed to find the slope of a line perpendicular to BC. Some candidates found the midpoint of BC and used it to find the equation.
Many candidates did not express the equation in the simplified form. In many scripts, calculation errors were also observed.

(c) Concept of Locus was not clear to some candidates.
Many candidates did not show the necessary traces of construction. Midpoint of BC was not located by construction instead used ruler. Some were unable to identify the point A as the intersection of perpendicular bisector and semi-circle.
A number of candidates failed to bisect ∠ABC to locate point D on the semicircle.

Suggestions for teachers

- Teach basic concepts of series in detail in the class.
- While taking square root of a number discuss the reason of taking values both with positive and negative signs unless otherwise specified in the question.
- Clarify the concept of slope of a line, equation of a line perpendicular or parallel to a given line in detail.
- Give rigorous practice on the content of coordinate geometry.
- Instruct students to be careful with positive and negative signs during simplification of equations.
- Give enough practice in problems based on geometry.
- Train students to practise constructions using ruler and compass only, unless otherwise specified in the question.
- Instruct students to show all necessary traces of construction clearly.

MARKING SCHEME

Question 8

(a) Let the terms be $a-d, a, a+d$

$\therefore a-d+a+a-d=42$

$3a=42$

$\therefore a=14$

$(a-d)(a+d)=52$

$14^2-d^2=52$

$d^2=196-52$

$d^2=144$

$\therefore d=\pm 12$

$d=12$, or -12

(b) A(3, 8), B(–1, 2) and C(6, –6)

(i) Slope of line BC $= \frac{-6-2}{6+1} = \frac{-8}{7}$

(ii) Slope of line perpendicular to BC is $\frac{7}{8}$; Line passing through A(3, 8)

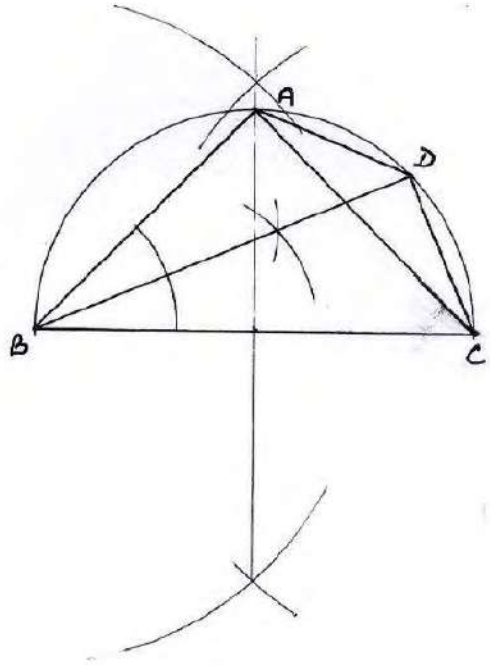
Equation is:

$y - 8 = \frac{7}{8}(x - 3)$

$8y - 64 = 7x - 21$

$7x - 8y + 43 = 0$

(c) $\angle ADC = 135°$



Question 9

(a) The data on the number of patients attending a hospital in a month are given below. Find the average (mean) number of patients attending the hospital in a month by using the shortcut method. [3]

Take the assumed mean as 45. Give your answer correct to 2 decimal places.

Number of patients	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Number of Days	5	2	7	9	2	5

(b) Using properties of proportion solve for x, given [3]

$$\frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}-\sqrt{2x-6}} = 4$$

(c) Sachin invests ₹8500 in 10%, ₹100 shares at ₹170. He sells the shares when the price of each share rises by ₹30. He invests the proceeds in 12% ₹100 shares at ₹125. Find: [4]

(i) the sale proceeds.

(ii) the number of ₹125 shares he buys.

(iii) the change in his annual income.

Comments of Examiners

(a) Following errors were observed in this question:
- Class mark was incorrect.
- Assumed mean was not taken as 45 as it was given in the question.
- Some candidates did not use the short-cut method to find the mean.
- A very common error was 121÷3 was written as 4.33 instead of 40.33.

(b) Many candidates made errors in applying Componendo and Dividendo especially in the denominator of Left-Hand Side. Some candidates applied properties of proportion only on one side.
Several candidates made mistake in squaring the expression $\frac{2\sqrt{5x}}{2\sqrt{2x-6}} = \frac{5}{3}$

(c) (i) Many candidates took sale price of the share as ₹130 instead of ₹200, Hence, answer to number of shares was incorrect. Calculation errors were also observed in many scripts.

(ii) Many candidates could not find the correct answer for number of shares he bought due to error in calculating the sale proceeds.

(iii) Error was also observed in the change in annual income due to incorrect value of number of shares.

Suggestions for teachers

- Advise students to read the question carefully and keep in mind the known and unknown data e.g., assumed mean =45, use short-cut method to find the mean, express your answer correct to 2 decimal places etc.
- Adequate practise of problems based on computation of mean must be given.
- Give more drill to avoid error in calculation.
- Revise frequently the problems on Ratio and Proportion.
- Show all steps of using properties of proportion in the solution of a problem clearly.
- Explain exhaustively the terms related to Share and Dividends-market value, face/nominal value, dividend etc thoroughly with examples.

MARKING SCHEME

Question 9

(a)

C.I	f	mid-value	d	fd
10 - 20	5	15	-30	-150
20 - 30	2	25	-20	-40
30 - 40	7	35	-10	-70
40 - 50	9	45	0	0
50 - 60	2	55	10	20
60 - 70	5	65	20	100
	30			-140

Given assumed mean (A) = 45

Mean = $45 + \left(\frac{-140}{30}\right)$

$= 45 - 4.67$

$= 40.33$

(b)

$$\frac{\sqrt{5x} + \sqrt{2x-6}}{\sqrt{5x} - \sqrt{2x-6}} = 4$$

$$\frac{\sqrt{5x}+\sqrt{2x-6}+\sqrt{5x}-\sqrt{2x-6}}{\sqrt{5x}+\sqrt{2x-6}-\sqrt{5x}+\sqrt{2x-6}} = \frac{4+1}{4-1}$$ Applying componendo and dividendo

$$\frac{2\sqrt{5x}}{2\sqrt{2x-6}} = \frac{5}{3}$$

Squaring both sides

$$\frac{5x}{2x-6} = \frac{25}{9}$$

$$45x = 50x - 150$$

$$5x = 150$$

$$\therefore\ x = 30$$

(c)

Number of shares = $\frac{8500}{170} = 50$

Dividend = $\frac{50\times100\times10}{100} = ₹500$

(i) Sales proceeds = $50 \times (170+30) = ₹10,000$

(ii) New number of shares = $\frac{10,000}{125} = 80$

Dividend from new shares = $\frac{80\times100\times12}{100} = ₹960$

$\therefore$ Change in income = $960 - 500 = ₹460$

Question 10

(a) Use graph paper for this question. [6]

The marks obtained by 120 students in an English test are given below:

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
No. of students	5	9	16	22	26	18	11	6	4	3

Draw the ogive and hence, estimate:

(i) the median marks.

(ii) the number of students who did not pass the test if the pass percentage was 50.

(iii) the upper quartile marks.

(b) A man observes the angle of elevation of the top of the tower to be 45°. He walks towards it in a horizontal line through its base. On covering 20 m the angle of elevation changes to 60°. Find the height of the tower correct to 2 significant figures. [4]

Comments of Examiners

(a) (i) A number of candidates made mistakes in finding the cumulative frequency though the total frequency was given correctly.
Some candidates drew the ogive with respect to lower boundaries and corresponding cumulative frequency instead of taking upper boundaries.
Some candidates drew the ogive taking mid-values and corresponding cumulative frequency.
To locate the various results from the graph indicating guidelines were not shown.
Several candidates formed the S-curve by joining the points with ruler instead of free hand curve.

In sub parts (ii) and (iii) some candidates made mistakes in finding median marks, upper quartile marks and number of students who did not pass the test.

(b) The errors observed in this question were as follows:

- Many candidates drew incorrect diagram.
- Some candidates wrote incorrect ratio for tan θ.

Suggestions for teachers

- Teach the students to cross-check the cumulative frequency found. This is by tallying the total frequency with the last value of C.F.
- Correct choice of axis and scale require additional attention.
- Ensure that sufficient practice is given in drawing Ogive.
- Instruct students to indicate on the graph sheet to locate values from the graph.
- Advise students to draw Ogive - a cumulative frequency curve as a free hand curve. Points must not be joined with a ruler.
- Give sufficient practice of drawing correct diagrams for problems based on Heights and Distances.

- Many calculation errors were observed in the scripts because of taking tan 60° =1.732 instead of $\sqrt{3}$.

In several answer scripts height of tower was not expressed correct to 2 significant figures as it was asked in the question.

- Instruct students to learn the skill to find T-ratios of standard angles mathematical tables instead of using.
- Advise students read the question carefully to give the answer in the required form.
- Advice students to practice mathematical calculations each day to reduce calculation errors.

MARKING SCHEME

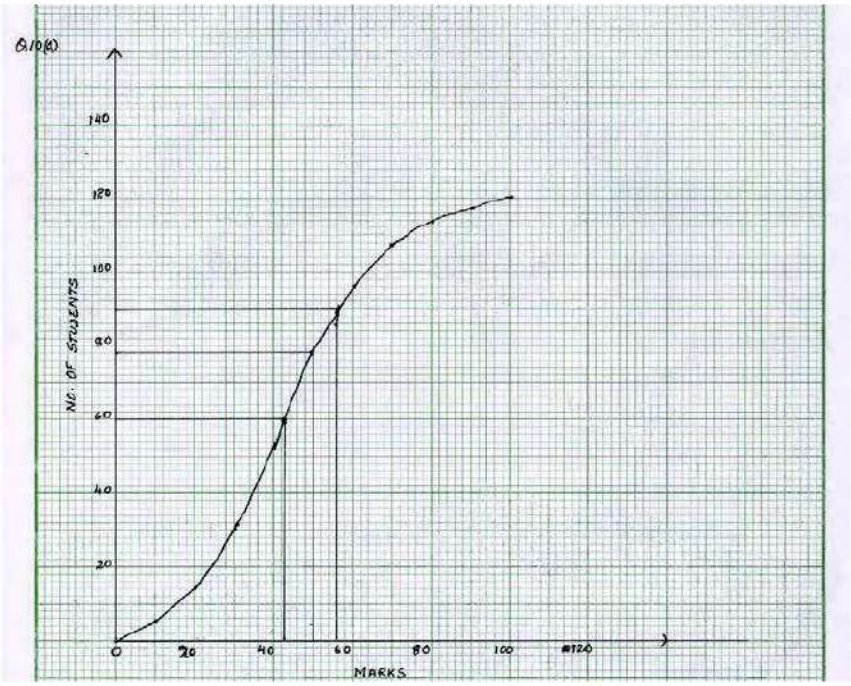
Question 10

(a)

C.I.	F	C.F.
0 – 10	5	5
10 – 20	9	14
20 – 30	16	30
30 – 40	22	52
40 – 50	26	78
50 – 60	18	96
60 – 70	11	107
70 – 80	6	113
80 – 90	4	117
90 – 100	3	120

SCALE: On X-axis, 1 cm = 20 marks
On Y-axis, 1 cm = 20 students

(i) Median = 43 marks
(ii) Number of students who did not pass the test 78
(iii) 56 marks (±1)



(b) $\tan 45^\circ = \frac{y}{x+20}$ $\{\tan 45^\circ = 1\}$

$1 = \frac{y}{x+20}$

$\therefore\ x + 20 = y$

$y - 20 = x$

$\tan 60 = \frac{y}{x}$ $\{\tan 60^\circ = \sqrt{3}\}$

$x\sqrt{3} = y$

$x = y/\sqrt{3}$

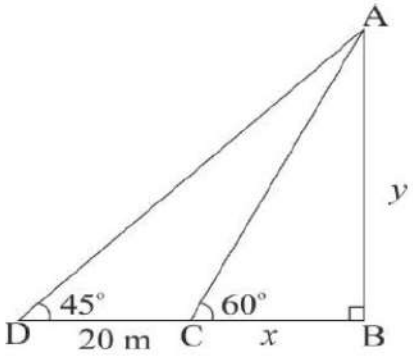
$y - 20 = y/\sqrt{3}$

$\therefore y = \frac{20(\sqrt{3})}{(\sqrt{3}-1)},\ y = \frac{20(\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$x = \frac{20(1.732+3)}{3-1}$

$x = 10 \times 4.732$

$= 47.32$ m



Question 11

(a) Using the Remainder Theorem find the remainders obtained when [3]

$x^3 + (kx + 8)x + k$ is divided by $x + 1$ and $x - 2$.

Hence find k if the sum of the two remainders is 1.

(b) The product of two consecutive natural numbers which are multiples of 3 is equal to 810. [3]

Find the two numbers.

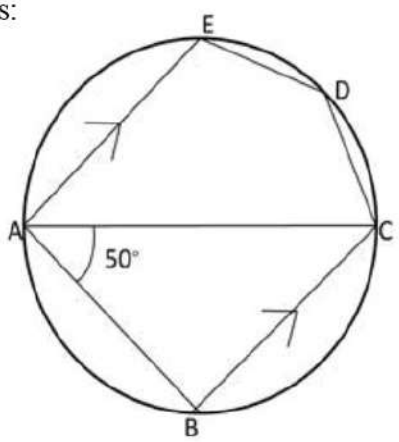
(c) In the given figure, ABCDE is a pentagon inscribed in a circle such that AC is a diameter and side BC//AE. If ∠BAC = 50°, find giving reasons: [4]

(i) ∠ACB

(ii) ∠EDC

(iii) ∠BEC

Hence prove that BE is also a diameter



Comments of Examiners

(a) Many candidates could not apply the Remainder theorem correctly.
Due to incorrect substitution or incorrect working some candidates found the incorrect remainder.
several candidates instead of adding the two remainders, equated each remainder to zero that is 5k+24 = 0 and 2k- 9 =0.

(b) The most common error noticed in several scripts was in taking two consecutive natural numbers which were multiples of 3.
Some candidates made mistakes in forming the equation with the given conditions.
Few candidates made errors in solving the quadratic equation.

(c) (i) Following errors were observed in this question:
- Some candidates could not apply the appropriate property of circle theorem to find out the unknown angles.

Suggestions for teachers

- Instruct students to read the question heedfully and analyse the given conditions before solving the problem.
- Stress upon solving sums using given conditions only.
- Give adequate practice in solving
  - problems based on Remainder-Factor theorem and
  - word problems based on quadratic equation.
- Clarify to the students about the consecutive numbers with different conditions like multiples of 3 with examples.
- Give ample practice to solve quadratic equation.

- Some failed to locate angles in the same segment hence could not prove ∠BEC=∠BAC=50°
- Properties of cyclic quadrilateral were not applied.
- Many candidates even after solving the other parts could not prove BE is a diameter giving proper reasons. Unable to find the values of $\angle ACB$, $\angle EDC$ and $\angle BEC$ giving appropriate reasons.

- Teach circles and related angle properties, cyclic properties, tangent and secant properties thoroughly.
- Train students to write correct reasons in support of the working of geometry based questions to obtain the answers.

MARKING SCHEME

Question 11

(a)

$x^3 + (kx + 8)x + k = x^3 + kx^2 + 8x + k$

When divided by $x + 1$

Remainder = $(-1)^3 + k(-1)^2 + 8(-1) + k$

$= -1 + k - 8 + k$

$= 2k - 9$

When divided by $x - 2$

Remainder = $(2)^3 + k(2)^2 + 8(2) + k$

$= 8 + 4k + 16 + k$

$= 5k + 24$

Hence $2k - 9 + 5k + 24 = 1$

$7k + 15 = 1$

$7k = -14$

$\therefore k = -2$

(b)

Assume that two consecutive natural numbers which are multiples of 3 *are* x *and* $x + 3$

$x(x + 3) = 810$

$x^2 + 3x = 810$

$x^2 + 3x - 810 = 0$

$(x + 30)(x - 27) = 0$

$x = -30$ *or* $x = 27$

therefore, the two numbers are 27 *and* 30.

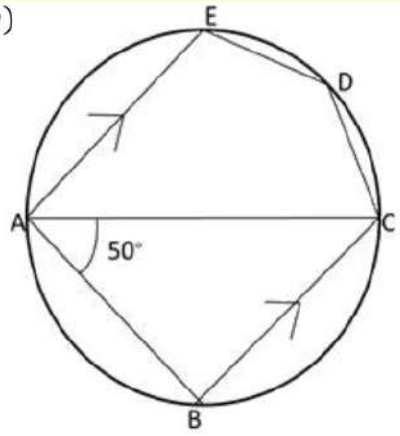
(c) (i) In $\Delta ABC, \angle ABC = 90°$ $(Angle\ in\ a\ semi\ circle)$

$\angle BAC = 50°$ $(given)$

$\therefore \angle ACB = 40°$

(ii) $\because$ AE//BC

$\therefore \angle CAE = \angle ACB = 40°$ $(pair\ of\ alternate\ \angle s)$



In cyclic quadrilateral ACDE

$\angle CAE + \angle EDC = 180°$ $(opposite\ \angle s\ of\ cyclic\ quadrilateral\ are\ supplementary.)$

$40° + \angle EDC = 180°$

$\angle EDC = 140°$

(iii) $\angle BEC = \angle BAC = 50°$ (angles in the same segment)

(iv) $\angle AEB = 90 - 50 = 40°$

$\therefore \angle EBC = 40°$ (alternate angles)

$\therefore \angle ECB = 90°$

$\therefore BE$ is a diameter.

Note: For questions having more than one correct answer/solution, alternate correct answers/solutions, apart from those given in the marking scheme, have also been accepted.

ICSE SOLVED PAPER - 2020

Class-X

Mathematics

(Two hours and a half)

Answers to this Paper must be written on the paper provided separately.
*You will **not** be allowed to write during the first **15** minutes.*
This time is to be spent in reading the question paper.
The time given at the head of this Paper is the time allowed for writing the answers.

---

*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*
All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.
Omission of essential working will result in loss of marks.
The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical tables are provided.

SECTION-A (40 Marks)

*Attempt **all** questions from this Section*

Question 1.

(a) Solve the following Quadratic Equation : **[3]**

$x^2 - 7x + 3 = 0$

Give your answer correct to two decimal places.

(b) Given $A = \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}$

If $A^2 = 3I$, where I is the identity matrix of order 2, find x and y. **[3]**

(c) Using ruler and compass construct a triangle ABC where $AB = 3$ cm, $BC = 4$ cm and $\angle ABC = 90°$. Hence construct a circle circumscribing triangle ABC. Measure and write down the radius of the circle. **[4]**

Question 2.

(a) Use factor theorem to factorise $6x^3 + 17x^2 + 4x - 12$ completely. **[3]**

(b) Solve the following inequation and represent the solution set on the number line. **[3]**

$\frac{3x}{5} + 2 < x + 4 \leq \frac{x}{2} + 5, x \in R$

(c) Draw a Histogram for the given data, using a graph paper :

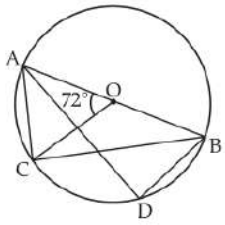
Weekly Wages (in ₹)	No. of People
3000 – 4000	4
4000 - 5000	9
5000 - 6000	18
6000 - 7000	6
7000 - 8000	7
8000 - 9000	2
9000 - 10000	4

Estimate the mode from the graph. **[4]**

Question 3.

(a) In the figure given below, O is the centre of the circle and AB is a diameter. **[3]**
If $AC = BD$ and $\angle AOC = 72°$ Find.

(i) $\angle ABC$
(ii) $\angle BAD$
(iii) $\angle ABD$



(b) Prove that : [3]

$$\frac{\sin A}{1+\cot A} - \frac{\cos A}{1+\tan A} = \sin A - \cos A$$

(c) In what ratio is the line joining $P(5, 3)$ and $Q(-5, 3)$ divided by the y-axis ? Also find the coordinates of the point of intersection. [4]

Question 4.

(a) A solid spherical ball of radius 6 cm is melted and recast into 64 identical spherical marbles. Find the radius of each marble. [3]

(b) Each of the letters of the word 'AUTHORIZES' is written on identical circular discs and put in a bag. They are well shuffled. If a disc is drawn at random from the bag, what is the probability that the letter is :

(i) a vowel

(ii) one of the fist 9 letters of the English alphabet which appears in the given word

(iii) one of the last 9 letters of the English alphabet which appears in the given word ? [3]

(c) Mr. Bedi visits the market and buys the following articles :

Medicines costing ₹ 950, GST @ 5%

A Pair of shoes costing ₹ 3000, GST @ 18%

A Laptop bag costing ₹ 1000 with a discount of 30% GST @ 18%

(i) Calculate the total amount of GST paid.

(ii) The total bill amount including GST paid by Mr. Bedi. [4]

SECTION-B (40 Marks)

*Attempt any **four** questions from this Section*

Question 5.

(a) A company with 500 shares of nominal value ₹ 120 declares an annual dividend of 15%. Calculate :

(i) the total amount of dividend paid by the company.

(ii) annual income of Mr. Sharma who holds 80 shares of the company. [3]

If the return percent of Mr. Sharma from his shares is 10%. Find the market value of each share.

(b) The mean of the following data is 16. Calculate the value of f. [3]

Marks	5	10	15	20	25
No. of Students	3	7	f	9	6

(c) The 4th, 6th and the last term of a geometric progression are 10, 40 and 640 respectively. If the common ratio is positive, find the first term, common ratio and the number of terms of the series. [4]

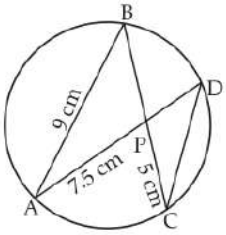
Question 6.

(a) If $A = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$ [3]

Find $A^2 - 2AB + B^2$.

(b) In the given figure $AB = 9$ cm, $PA = 7.5$ cm and $PC = 5$ cm. [3]

Chords AD and BC intersect at P.



(i) Prove that $\Delta PAB \sim \Delta PCD$

(ii) Find the length of CD.

(iii) Find area of ΔPAB : area of ΔPCD

(c) From the top of a cliff, the angle of depression of the top and bottom of a tower are observed to be 45° and 60° respectively. If the height of the tower is 20 m. **[4]**

Find :

(i) the height of the cliff

(ii) the distance between the cliff and the tower.

Question 7.

(a) Find the value of 'p' if the lines, $5x - 3y + 2 = 0$ and $6x - py + 7 = 0$ are perpendicular to each other. Hence, find the equation of a line passing through $(-2, -1)$ and parallel to $6x - py + 7 = 0$. **[3]**

(b) Using properties of proportion find $x : y$, given : **[3]**

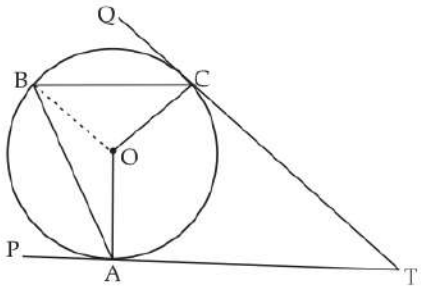
$$\frac{x^2 + 2x}{2x + 4} = \frac{y^2 + 3y}{3y + 9}$$

(c) In the given figure TP and TQ are two tangents to the circle with centre O, touching at A and C respectively. If $\angle BCQ = 55°$ and $\angle BAP = 60°$, find : **[4]**

(i) $\angle OBA$ and $\angle OBC$

(ii) $\angle AOC$

(iii) $\angle ATC$



Question 8.

(a) What must be added to the polynomial $2x^3 - 3x^2 - 8x$, so that it leaves a remainder 10 when divided by $2x + 1$? **[3]**

(b) Mr. Sona has a recurring deposit account and deposits ₹ 750 per month for 2 years. If he gets ₹ 19125 at the time of maturity, find the rate of interest. **[3]**

(c) Use graph paper for this question. **[4]**

Take 1 cm = 1 unit on both x and y axes.

(i) Plot the following points on your graph sheets.
A (–4, 0), B (–3, 2), C (0, 4), D (4, 1) and E (7, 3)

(ii) Reflect the points B, C, D and E on the x-axis and name them as B', C', D' and E' respectively

(iii) Join the points A, B, C, D, E, E', D', C', B' and A in order.

(iv) Name the closed figure formed.

Question 9.

(a) 40 Students enter for a game of shot-put competition. The distance thrown (in metres) is recorded below :

Distance in m	12 – 13	13 – 14	14 – 15	15 – 16	16 – 17	17 – 18	18 – 19
Number of Students	3	9	12	9	4	2	1

Use a graph paper to draw an ogive for the above distribution.

Use a scale of 2 cm = 1 m on one axis and 2 cm = 5 students on the other axis.

Hence using your graph find :

(i) the median

(ii) Upper Qartile

(iii) Number of students who cover a distance which is above $16\frac{1}{2}$ m. **[6]**

(b) If $x = \dfrac{\sqrt{2a+1} + \sqrt{2a-1}}{\sqrt{2a+1} - \sqrt{2a-1}}$, prove that $x^2 - 4ax + 1 = 0$. **[4]**

Question 10.

(a) If the 6th term of an A.P. is equal to four times its first term and the sum of fist six terms is 75, find the first term and the common difference. **[3]**

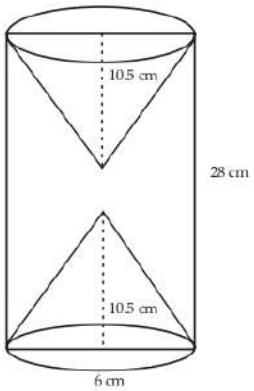
(b) The difference of two natural numbers is 7 and their product is 450. **[3]**
Find the numbers.

(c) Use ruler and compass for this question. Construct a circle of radius 4.5 cm. Draw a chord. $AB = 6$ cm.
(i) Find the locus of points equidistant from A and B.
Mark the point where it meets the circle as D.
(ii) Join AD and find the locus of points which are equidistant from AD and AB. Mark the point where it meets the circle as C.
(iii) Join BC and CD, Mesuare and write down the length of side CD of the quadrilateral $ABCD$. **[4]**

Question 11.

(a) A model of a high rise building is made to a scale of 1 : 50 **[3]**
(i) If the height of the model is 0.8 m, find the height of the actual building.
(ii) If the floor area of a flat in the building is 20 m^2, find the floor area of that in the model.

(b) From a solid wooden cylinder of height 28 cm and diameter 6 cm, two conical cavities are hollowed out. The diameters of the cones are also of 6 cm and height 10.5 cm.
Taking $\pi = \frac{22}{7}$ find the volume of the remaining solid.



[3]

(c) Prove the identity
$$\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \tan^2\theta$$ **[4]**

Solutions

SECTION-A (40 Marks)

*Attempt **all** questions from this Section*

Solution 1.

(a) $x^2 - 7x + 3 = 0$

Compare the equation by $ax^2 + bx + c = 0$
then, $a = 1, b = -7$ and $c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{7 \pm \sqrt{49 - 4\times1\times3}}{2\times1}$$

$$= \frac{7 \pm \sqrt{37}}{2}$$

$$= \frac{7 \pm \sqrt{37}}{2} = \frac{7 \pm 6.08}{2}$$

$\therefore$ $$x = \frac{7+6.08}{2} = \frac{13.08}{2} = 6.54$$

and $$x = \frac{7-6.08}{2} = \frac{0.92}{2} = 0.46$$

Hence $x = 6.54$ (Approx.) and $x = 0.46$ (Approx)

(b) $$A = \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}$$

$$A^2 = A \times A$$

$$= \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}\begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}$$

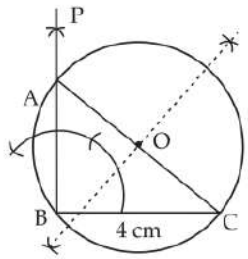
$$= \begin{bmatrix} x^2+3y & 3x+9 \\ xy+3y & 3y+9 \end{bmatrix}$$

$A^2 = 3I$ (Given)

$$\begin{bmatrix} x^2+3y & 3x+9 \\ xy+3y & 3y+9 \end{bmatrix} = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Comparing the elements of matrices
$3x + 9 = 0 \Rightarrow x = -3$
$3y + 9 = 3 \Rightarrow y = -2$

(c)



Steps of construction
(i) Draw $BC = 4$ cm
(ii) make $\angle PBC = 90°$
(iii) taking centre B and radius 3 cm draw a arc which intersect BP at the point A
(iv) Join AC, ΔABC is required triangle $\angle B = 90°$
$\therefore$ AC is the diameter of circle.
So find out mid point of AC
(v) Draw perpendicular or bisector of AC.
(vi) Taking centre O and radius equal to BO draw a circle which passes through vertex A, B and C.
Radius of circumscribe circle = 2.5 cm.

Solution 2.

(a) $p(x) = 6x^3 + 17x^2 + 4x - 12$
Remainder $p(-2) = 6(-2)^3 + 17(-2)^2 + 4(-2) - 12$
$= -48 + 68 - 8 - 12$

$= 68 - 68 = 0$

$\therefore (x + 2)$ is a factor of given polynomial $p(x)$

$$x+2\overline{)6x^3+17x^2+4x-12}\,(6x^2+5x-6$$

$6x^3 + 12x^2$
$(-)\quad(-)$
$5x^2 + 4x - 12$
$5x^2 + 10x$
$(-)\quad(-)$
$-6x - 12$
$-6x - 12$
$(+)\quad(+)$
0

$\therefore 6x^3 + 17x^2 + 4x - 12$

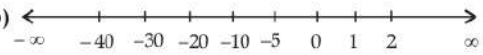
$= (x + 2)(6x^2 + 5x - 6)$

$= (x + 2)\{6x^2 + 9x - 4x - 6\}$

$= (x + 2)\{3x(2x + 3) - 2(2x + 3)\}$

$= (x + 2)(2x + 3)(3x - 2)$

(b)



$$\frac{3x}{5}+2<x+4\le\frac{x}{2}+5,\quad x\in R$$

$$\frac{3x}{5}+2<x+14$$

$$\frac{3x}{5}-x<14-2$$

$$\frac{3x-5x}{5}<12$$

Multiply by 5

$$-2x<12\times5$$

$$\therefore\ x>\frac{60}{-2}$$

$$\therefore\ x>-30$$

OR

$$x+4\le\frac{x}{2}+5$$

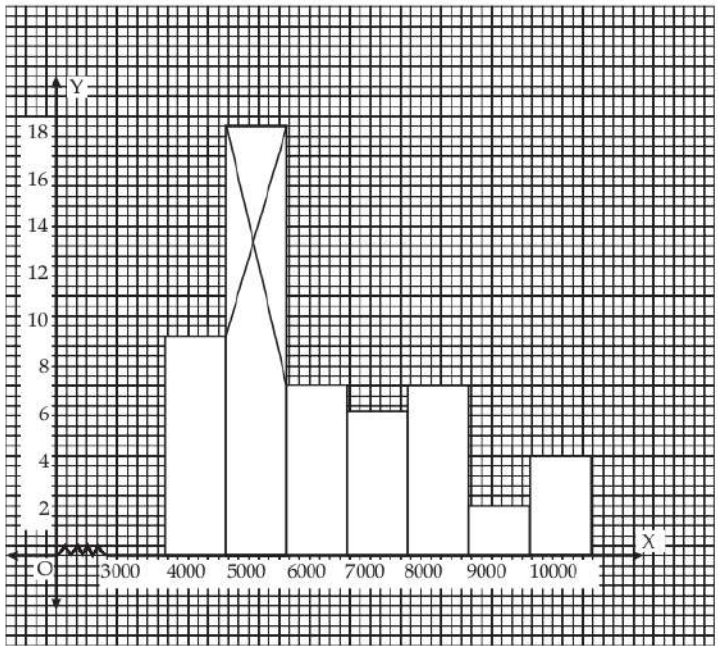
$$x-\frac{x}{2}\le5-4$$

$$\frac{x}{2}\le1$$

$$x\le2$$

$$\therefore\ -30<x\le2$$

(c)



Solution 3.

(a) AB is the diameter of circle

$\therefore \quad \angle ADB = 90°$ (semicircle angle)

(i) $\angle ABC = \frac{1}{2}\angle AOC$ (same arc angles at circumference and centres)

$\therefore \quad \angle ABC = \frac{1}{2}\times 72 = 36°$

(ii) $AC = BD$

$\therefore \quad \angle ABC = \angle BAD$ (equal arcs make equal angle)

$\therefore \quad \angle BAD = 36°$

(iii) In ΔABD

$\angle ABD + \angle BAD + \angle ADB = 180°$ (sum of all angles of Δ)

$\angle ABD + 36° + 90° = 180°$

$\angle ABD = 180° - 126°$

$\angle ABD = 54°$

(b) $\dfrac{\sin A}{1+\cot A}-\dfrac{\cos A}{1+\tan A}=\sin A-\cos A$

$$\text{LHS} = \frac{\sin A}{1+\cot A} - \frac{\cos A}{1+\tan A}$$

$$= \frac{\sin A}{1+\frac{\cos A}{\sin A}} - \frac{\cos A}{1+\frac{\sin A}{\cos A}}$$

$$= \frac{\sin^2 A}{\sin A+\cos A} - \frac{\cos^2 A}{\cos A+\sin A}$$

$$= \frac{\sin^2 A-\cos^2 A}{\sin A+\cos A}$$

$$[a^2-b^2 = (a+b)(a-b)]$$

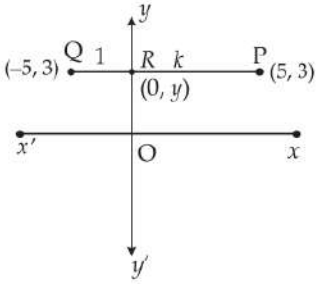
$$= \frac{(\sin A+\cos A)(\sin A-\cos A)}{(\sin A+\cos A)}$$

$$= \sin A - \cos A$$

$$= \text{RHS.}$$

Hence proved.

(c) Let the line joining the (5, 3) and (– 5, 3) divided by the y-axis in the ratio $k : 1$.



Let the coordinate of the point of intersection R be $(0, y)$.

$$R_x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$$

$$0 = \frac{1\times 5+k(-5)}{1+k}$$

$$5-5k = 0 \Rightarrow k = 1$$

$$R_y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$$

$$y = \frac{1\times 3+k\times 3}{1+k}$$

$$y = \frac{3+3}{1+1} = 3 \quad [\text{Put, k= 1}]$$

$$\text{Ratio} = k : 1 = 1 : 1$$

Point of intersection of the line (0, 3)

Solution 4.

(a) Volume of solid spherical ball $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi(6)^3$$

$$= \frac{4}{3}\pi\times 216$$

$$= \frac{216\times 4}{3}\pi \text{ cm}^3$$

Let the radius of spherical marble be R cm.

Then 64 × volume of spherical marbles = volume of spherical ball

$$64\times\frac{4}{3}\pi R^3 = \frac{4\times 216}{3}\pi \text{ cm}^3$$

$$R^3 = \frac{216}{64}$$

$$R = \sqrt[3]{\frac{216}{64}} = \frac{6}{4} = \frac{3}{2} \text{ cm}$$

$$= 1.5 \text{ cm}$$

$\therefore$ Radius of spherical marble is 1.5 cm

(b) 'AUTHORIZES'

Total number of disc $n(s) = 10$

(i) number of discs a letter of vowel = 5

(A, U, O, I, E) $\therefore n(E) = 5$

Probability of a disc is vowel

$$= \frac{n(E)}{n(s)} = \frac{5}{10} = \frac{1}{2}$$

(ii) one of the first 9 letter in english alphabet

$= 4$ (A, E, H, I)

$\therefore \quad n(E) = 4$

Probability of a disc is drawn is written one of first 9 letters of english alphabet

$$= \frac{n(E)}{n(s)} = \frac{4}{10}$$

$$= \frac{2}{5}$$

(iii) One of the last 9 (R, S, T, U, Z) letters of english alphabet

$n(E) = 5$

Probability of a disc is drawn is written one of the last 9 letters of english alphabet

$$= \frac{n(E)}{n(s)} = \frac{5}{10}$$

$$= \frac{1}{2}$$

(c) Medicine costing = ₹ 950

$$\text{GST @ 5\%} = \frac{5}{100}\times 950 = ₹\ 47.50$$

A pair of shoes costing = ₹ 3000

$$\text{GST @ 18\%} = \frac{18}{100}\times 3000 = ₹\ 540$$

Laptop bag costing ₹ 1000 with a discount of 30%

$$\therefore \quad \text{Net cost} = 1000 - \frac{30}{100}\times 1000$$

$$= ₹\ 700$$

$$\text{GST @ 18\%} = \frac{18}{100}\times 700 = ₹\ 126$$

(i) Total amount of GST = ₹ 47.50 + 540 + 126

= ₹ 713.50

(ii) Total Bill amount including GST

= ₹ 950 + 3000 + 700 + 713.50

= ₹ 4650 + 713.50

= ₹ 5363.50

SECTION-B (40 Marks)

Solution 5.

(a) Number of shares = 500
Nominal value = ₹ 120
Annual dividend = 15%

(i) Total amount of dividend paid by the company = Rate of dividend × Nominal value of share × Number of share

$$= \frac{15}{100} \times 120 \times 500$$

$$= 15 \times 600$$

∴ Total amount paid by the company = ₹ 9000

(ii) Mr sharma's Annual Income = Rate of dividend × Nominal value of share × Number of share

$$= \frac{15}{100} \times 120 \times 80$$

$$= 15 \times 96$$

∴ Mr sharma's Annual Income = ₹ 1440

$$\text{Return of Investment} = \frac{\text{Income}}{\text{Investment}} \times 100$$

$$10 = \frac{1440}{\text{Investment}} \times 100$$

$$\therefore \text{ Investment} = \frac{144000}{10} = ₹\ 14400$$

$$\text{Market value of share} = \frac{\text{Investment}}{\text{No. of share}}$$

$$= \frac{14400}{80}$$

∴ Market value of share = ₹ 180

(b)

Marks	No. of Students	
(x)	(f)	$f \times x$
5	3	15
10	7	70
15	f	$15f$
20	9	180
25	6	150
	$25 + f$	$415 + 15f$

Mean = 16 (Given)

$$\frac{\Sigma fx}{\Sigma f} = 16$$

$$\frac{415 + 15f}{25 + f} = 16$$

$$415 + 15f = 400 + 16f$$

$$16f - 15f = 415 - 400$$

$$\therefore \quad f = 15$$

(c) Let the first term of G.P. be a, common ratio be r and number of terms be n

$a_4 = 10$, $a_6 = 40$ and $a_n = 640$ (given)

$$a_4 = 10$$

$$ar^3 = 10 \quad \text{...(i)}$$

Again, $a_6 = 40$

$$ar^5 = 40 \quad \text{...(ii)}$$

from (i) & (ii)

$$\frac{ar^5}{ar^3} = \frac{40}{10}$$

$$\Rightarrow \quad r^2 = 4$$

$$\therefore \quad r = \pm 2 \quad (r > 0)$$

$$\therefore \quad r = 2$$

from equ. (i)

$$ar^3 = 10$$

$$a(2)^3 = 10$$

$$a = \frac{10}{8} = \frac{5}{4}$$

Again, $a_n = 640$

$$ar^{n-1} = 640$$

$$\frac{5}{4}(2)^{n-1} = 640$$

$$2^{n-1} = \frac{640 \times 4}{5}$$

$$2^{n-1} = 128 \times 4 = 2^9$$

Comparing the power

$$n - 1 = 9 \Rightarrow n = 10$$

∴ first term of G.P. = $\frac{5}{4}$, common ratio = 2

and number of terms = 10

Solution 6.

(a) $A = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$

$$A^2 = A \times A = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+0 & 0+0 \\ 15+5 & 0+1 \end{bmatrix}$$

$$\therefore \quad A^2 = \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -12+0 & 6+0 \\ -20+1 & 10+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} -12 & 6 \\ -19 & 10 \end{bmatrix}$$

$$B^2 = B \times B = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} +16+2 & -8+0 \\ -4+0 & 2+0 \end{bmatrix}$$

$$\therefore \quad B^2 = \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

$\therefore \quad A^2 - 2AB + B^2$

$$\begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix} - 2\begin{bmatrix} -12 & 6 \\ -19 & 10 \end{bmatrix} + \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix} + \begin{bmatrix} 24 & -12 \\ 38 & -20 \end{bmatrix} + \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

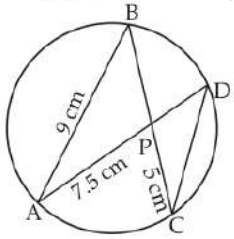
$$\begin{bmatrix} 9+24+18 & 0-12-8 \\ 20+38-4 & 1-20+2 \end{bmatrix} = \begin{bmatrix} 51 & -20 \\ 54 & -17 \end{bmatrix}$$

(b) (i) ΔPAB and ΔPCD

$\angle APB = \angle CPD$ (Vertically opposite angle)

$\angle ABP = \angle PDC$ (same arc $\overset{\frown}{AC}$ angles)

$\therefore \quad \Delta PAB \sim \Delta PCD$ (AA similarly test)



(ii) $$\frac{PA}{PC} = \frac{AB}{CD} = \frac{PB}{PD}$$

(Property of similar triangle)

$$\frac{PA}{PC} = \frac{AB}{CD}$$

$$\frac{7.5}{5} = \frac{9}{CD}$$

$$\therefore \quad CD = \frac{9\times 5}{7.5} = \frac{9\times 5\times 10}{75}$$

$$\therefore \quad CD = 6 \text{ cm}$$

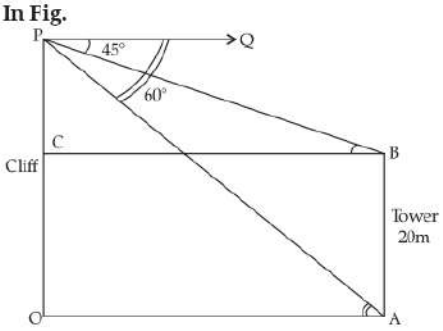
(iii) $$\frac{ar\Delta PAB}{ar\Delta PCD} = \frac{AP^2}{CP^2} = \left(\frac{AP}{CP}\right)^2$$

$$= \left(\frac{7.5}{5}\right)^2$$

$$= \frac{9}{4}$$

$\therefore \text{ar}\Delta PAB : \text{ar}\Delta PCD = 9 : 4$

(c) In Fig.



OP is the cliff and AB is the tower

In $\Delta PBC \qquad \tan B = \frac{CP}{BC}$

$$\tan 45° = \frac{CP}{BC}$$

$$1 = \frac{CP}{BC}$$

$\therefore \quad CP = BC$ ...(i)

In ΔOAP

$$\tan A = \frac{OP}{OA}$$

$(\because \angle QPA = \angle PAO = 60°)$

$$\tan 60° = \frac{OC+CP}{OA}$$

$$\sqrt{3} = \frac{AB+CP}{OA}$$

$(\because OC = AB = 20 \text{ m})$

$$\sqrt{3} = \frac{20+CP}{BC} \qquad (OA = BC)$$

$$\sqrt{3} = \frac{20+CP}{CP} \qquad (BC = CP)$$

$$\sqrt{3}\,CP = 20 + CP$$

$$\left(\sqrt{3}-1\right)CP = 20$$

$$CP = \frac{20}{\sqrt{3}-1}\text{m}$$

$$CP = \frac{20}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$CP = \frac{20(\sqrt{3}+1)}{3-1}$$

$$= 10(\sqrt{3}+1)\text{ m}$$

$$CP = 10\,(1.732 + 1)\text{m}$$

$$CP = 27.32 \text{ m}$$

(i) height of the cliff

$$OP = OC + CP$$

$$= 20 + 27.32$$

$\therefore$ Height of the cliff $= 47.32$ m

(ii) Distance between the cliff and the tower

$$OA = BC = CP$$

$$OA = 27.32 \text{ m}$$

Solution 7.

(a) Slop of line $5x - 3y + 2 = 0$ is

$$= -\frac{\text{coefficent of } x}{\text{coefficent of } y}$$

$$= \frac{-5}{-3} = \frac{5}{3}$$

Slope of line $6x - py + 7 = 0$ is $= \frac{-6}{-p} = \frac{6}{p}$

Lines are perpendicular each other

$\therefore$ Product of slops $= -1$

$$\frac{5}{3}\times\frac{6}{p} = -1$$

$$3p = -30 \Rightarrow p = -10$$

$\therefore$ $p = -10$

Given line $6x - py + 7 = 0$ is $6x + 10y + 7 = 0$

Equation of the line, parallel to $6x + 10y + 7 = 0$ is $6x + 10y + k = 0$

line passes through $(-2, -1)$

$\therefore 6(-2) + 10(-1) + k = 0$

$-12 - 10 + k = 0 \Rightarrow k = 22$

Equation of the line parallel to $6x + 10y + 7$ is

$6x + 10y + 22 = 0$ or $3x + 5y + 11 = 0$

(b) $$\frac{x^2+2x}{2x+4} = \frac{y^2+3y}{3y+9}$$

(using compodendo dividendo $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$)

$$\frac{x^2+2x+2x+4}{x^2+2x-2x-4} = \frac{y^2+3y+3y+9}{y^2+3y-3y-9}$$

$$\frac{x^2+4x+4}{x^2-4} = \frac{y^2+6y+9}{y^2-9}$$

$$\frac{(x+2)^2}{(x+2)(x-2)} = \frac{(y+3)^2}{(y+3)(y-3)}$$

$$\frac{x+2}{x-2} = \frac{y+3}{y-3}$$

(using compodendo - dividendo)

$$\frac{x+2+x-2}{x+2-x+2} = \frac{y+3+y-3}{y+3-y+3}$$

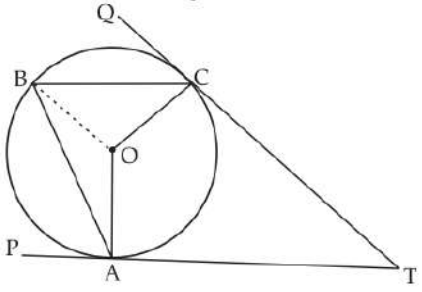
$$\frac{2x}{4} = \frac{2y}{6}$$

$$\frac{x}{2} = \frac{y}{3}$$

$$\frac{x}{y} = \frac{2}{3}$$

$\therefore$ $x : y = 2 : 3$

(c) Given, $\angle BCQ = 55°$ and $\angle BAP = 60°$



(i) $\because$ $\angle PAO = 90°$ (As PAT is the tangent)

$\therefore$ $\angle OAB = 90° - 60° = 30°$

In ΔOAB, $OA = OB$ ..... radii of circle

$\angle OBA = \angle OAB = 30°$

Similarly $\therefore \angle QCO = 90°$ (As QCT is the tangent)

$\therefore$ $\therefore \angle OCB = 90° - 55° = 35°$

In ΔOCB, $OC = OB$ ..... radii of circle

$\angle OBC = \angle OCB = 35°$

(ii) Now, $\angle ABC = \angle OBA + \angle OBC$

$= 30° + 35° = 65°$

Again, $\angle AOC = 2\angle ABC = 2 \times 65° = 130°$

($\because$ Again subtended at the center of the circle by an arc is twice the angle subtended at the circle)

(iii) $\angle AOC + \angle ATC = 180°$

$130° + \angle ATC = 180°$

$\therefore$ $\angle ATC = 180° - 130° = 50°$

$\therefore$ $\angle ATC = 50°$

Solution 8.

(a) $p(x) = 2x^3 - 3x^2 - 8x$

$q(x) = 2x + 1$

Remainder $= 10$

Let k be added to get remainder 10 when divided by $(2x + 1)$

$\therefore$ $p(x) = 2x^2 - 3x^2 - 8x + k$

Remainder $= 10$

$$p\left(-\frac{1}{2}\right) = 10$$

$$2\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 - 8\left(-\frac{1}{2}\right) + k = 10$$

$$-2\times\frac{1}{8} - 3\times\frac{1}{4} + 4 + k = 10$$

$$-1 + 4 + k = 10$$

$$k = 7$$

Hence 7 be added to get remainder 10 when divided by $2x + 1$ to given polynomial.

(b) Maturity amount (M.A.) = ₹ 19125/-

Monthly deposit (P) = ₹ 750/-

Time (n) = 2 years = 24 months

$$\text{M.A.} = P \times n + p \times \frac{r}{100} \times \frac{n(n+1)}{2} \times \frac{1}{12}$$

$$750 \times 24 + \frac{750\times r}{2400}(24)(24+1) = 19125$$

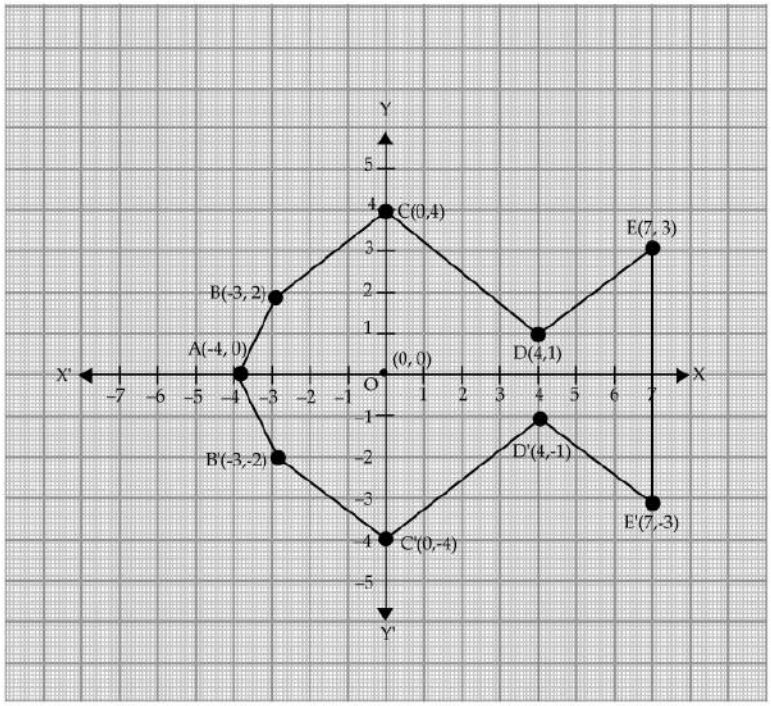
$$18000 + \frac{750}{4}r = 19125$$

$$\frac{750r}{4} = 19125 - 18000$$

$$r = \frac{1125\times 4}{750} = \frac{4500}{750}$$

$$r = 6$$

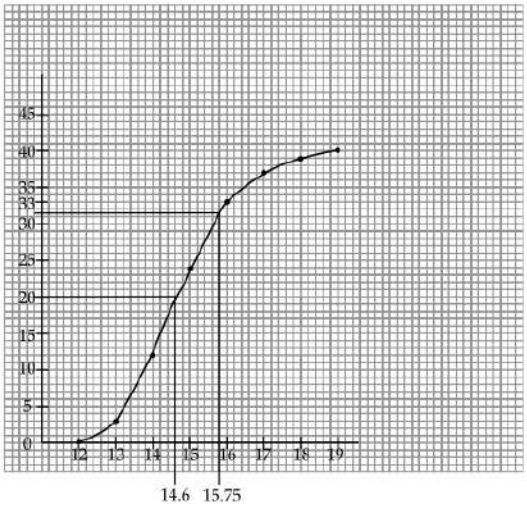
$\therefore$ rate of interest = 6%



(iv) closed figure formed the shape of fish

Solution 9.

(a)



Distance in (m)	Number of students	Less than	Number of persons.
12 - 13	3	12	0
13 -14	9	13	3
14 - 15	12	14	12
15 - 16	9	15	24
16 - 17	4	16	33
17 - 18	2	17	37
18 - 19	1	18	39
		19	40

(i) Median = 14.6

(ii) Upper quartile = 15.75

(iii) Number of students above $16\frac{1}{2}\,m = 6$

(b) $x = \dfrac{\sqrt{2a+1}+\sqrt{2a-1}}{\sqrt{2a+1}-\sqrt{2a-1}}$

$$\frac{x+1}{x-1} = \frac{\sqrt{2a+1}+\sqrt{2a-1}+\sqrt{2a+1}-\sqrt{2a-1}}{\sqrt{2a+1}+\sqrt{2a-1}-\sqrt{2a+1}+\sqrt{2a-1}}$$

(Apply compodendo-dividendo $\dfrac{A}{B}=\dfrac{C}{D}$

$\Rightarrow \dfrac{A+B}{A-B}=\dfrac{C+D}{C-D}$)

$$\frac{x+1}{x-1} = \frac{2\sqrt{2a+1}}{2\sqrt{2a-1}} = \frac{\sqrt{2a+1}}{\sqrt{2a-1}}$$

Squaring both sides

$$\frac{(x+1)^2}{(x-1)^2} = \left(\frac{\sqrt{2a+1}}{\sqrt{2a-1}}\right)^2$$

$$\frac{x^2+2x+1}{x^2-2x+1} = \frac{2a+1}{2a-1}$$

Apply compodendo - dividendo

$$\frac{x^2+2x+1+x^2-2x+1}{x^2+2x+1-x^2+2x-1} = \frac{2a+1+2a-1}{2a+1-2a+1}$$

$$\frac{2(x^2+1)}{4x} = \frac{4a}{2}$$

$$\frac{x^2+1}{2x} = \frac{2a}{1}$$

$$x^2 + 1 = 4ax$$

$$x^2 - 4ax + 1 = 0$$ **Hence proved.**

Solution 10.

(a) Let the first term of an A.P. be a and common difference be d respectively

$a_6 = 4 \times a$ (given)

$a_6 = 4a$ ...(i)

$a + 5d = 4a$ $\{a_n = a + (n-1)d\}$

$3a = 5d$

$a = \frac{5}{3}d$ ...(ii)

$S_6 = 75$ (given)

$\frac{n}{2}[a + a_6] = 75$

$\frac{6}{2}(a + a_6) = 75$

$a + a_6 = \frac{75}{3}$

$a + 4a = 25$

$a = \frac{25}{5} = 5$

$\therefore \quad a = 5$

from equation (i)

$a = \frac{5}{3}d$

$5 = \frac{5}{3}d$

$\therefore \quad d = \frac{5 \times 3}{5} \times 3$

$\therefore \quad d = 3$

first term of an A.P. = 5 and common difference = 3

(b) Let the numbers be x and y

According the given condition

$x - y = 7 \Rightarrow y = x - 7$ ...(i)

$xy = 450$ ...(ii)

from (i) & (ii)

$x(x-7) = 450$

$x^2 - 7x - 450 = 0$

$x^2 - 25x + 18x - 450 = 0$

$x(x-25) + 18(x-25) = 0$

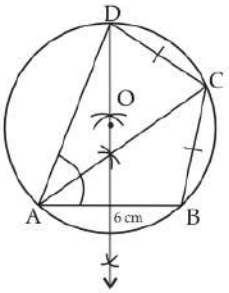
$(x-25)(x+18) = 0$

if $x + 18 = 0 \Rightarrow x = -18$ it is not natural no.

if $x - 25 = 0 \Rightarrow x = 25$, $y = 25 - 7 = 18$

Numbers 25 and 18

(c)



$BC = CD = 5.1$ cm (Approx.)

Solution 11.

(a) Scale = 1 : 50

height of the model 0.8 m

$\therefore$ height of the building = 0.8×50 m = 40 m

Floor area of building = $20 m^2$

Floor area of the model = $\frac{20}{50 \times 50} m^2$

$= \frac{1}{125} m^2$

$= 0.008\ m^2$

$\therefore$ Floor area of the model

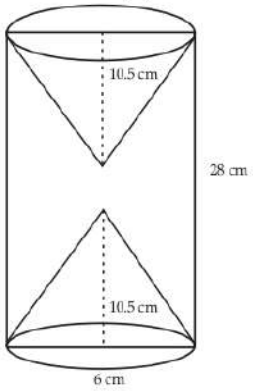
$= 0.008 \times 10000\ cm^2$

$= 80\ cm^2$

(b) Dimension of cylinder

Height (h) = 28 cm

diameter (2r) = 6 cm



Dimension of conical cavities

Height (H) = 10.5 cm

diameter (2r) = 6 cm

volume of remaining solid = volume of cylinder – 2 × volume of cavity

$= \pi r^2 h - 2 \times \frac{1}{3}\pi r^2 H$

$= \pi r^2 \left(h - \frac{2}{3}H\right)$

$= \frac{22}{7} \times 3 \times 3 (28 - \frac{2}{3} \times 10.5)$

$$= \frac{22}{7} \times 3 \times 3(28-7)$$

$$= \frac{22}{7} \times 3 \times 3 \times 21$$

$$= 22 \times 3 \times 3 \times 3$$

volume of remaining solid $= 594\ cm^3$

(c) $\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \tan^2\theta$

$$LHS = \left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2$$

$$= \left(\frac{1-\tan\theta}{1-\frac{1}{\tan\theta}}\right)^2$$

$$= \left(\frac{1-\tan\theta}{\frac{\tan\theta-1}{\tan\theta}}\right)^2$$

$$= \left\{\frac{\tan\theta\,(1-\tan\theta)}{-(1-\tan\theta)}\right\}^2$$

$$= \left\{\frac{\tan\theta}{-1}\right\}^2 \tan^2\theta$$

$= RHS$ **Hence proved.**

❑❑

ICSE SEMESTER-1 EXAMINATION

MATHEMATICS

Solved Paper - 2021-22

Class-10th

You will not be allowed to write during the first 10 minutes.

This time is to be spent in reading the question paper.

All questions are compulsory.

The marks intended for questions are given in brackets [ ].

Select the correct option for each of the following questions.

Max. Marks : 40 **Time allowed : 1:30 Hours**

Question 1.

If $(x+2)$ is a factor of the polynomial $x^3 - kx^2 - 5x + 6$ then the value of k is: [1]

(a) 1 **(b)** 2 **(c)** 3 **(d)** –2

Question 2.

The solution set of the inequation $x - 3 \geq -5, x \in R$ is: [1]

(a) $\{x : x > -2, x \in R\}$ **(b)** $\{x : x \leq -2, x \in R\}$

(c) $\{x : x \geq -2, x \in R\}$ **(d)** $\{-2, -1, 0, 1, 2\}$

Question 3.

The product AB of two matrices A and B is possible if: [1]

(a) A and B have the same number of rows.

(b) the number of columns of A is equal to the number of rows of B.

(c) the number of rows of A is equal to the number of columns of B.

(d) A and B have the same number of columns.

Question 4.

If 70, 75, 80, 85 are the first four terms of an Arithmetic Progression, then the 10^{th} term is: [1]

(a) 35 **(b)** 25 **(c)** 115 **(d)** 105

Question 5.

The selling price of a shirt excluding GST is ₹ 800. If the rate of GST is 12% then the total price of the shirt is: [1]

(a) ₹704 **(b)** ₹96 **(c)** ₹896 **(d)** ₹848

Question 6.

Which of the following quadratic equations has 2 and 3 as its roots? [1]

(a) $x^2 - 5x + 6 = 0$ **(b)** $x^2 + 5x + 6 = 0$

(c) $x^2 - 5x - 6 = 0$ **(d)** $x^2 + 5x - 6 = 0$

Question 7.

If $x, 5.4, 5, 9$ are in proportion then x is: [1]

(a) 3 **(b)** 9.72 **(c)** 25 **(d)** 25/3

Question 8.

Mohit opened a Recurring deposit account in a bank for 2 years. He deposits ₹1000 every month and receives ₹25500 on maturity. The interest he earned in 2 years is: [1]

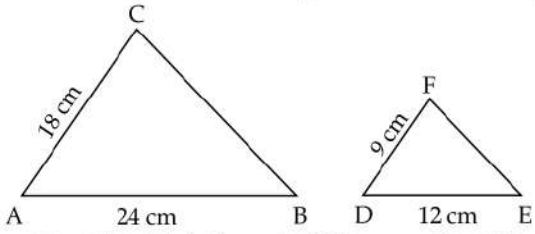
(a) ₹13500 **(b)** ₹3000

(c) ₹24000 **(d)** ₹1500

Question 9.

In the given figure $AB = 24$ cm, $AC = 18$ cm, $DE = 12$ cm, $DF = 9$ cm and $\angle BAC = \angle EDF$.

Then $\Delta ABC \sim \Delta DEF$ by the condition: [1]



(a) *AAA* **(b)** *SAS* **(c)** *SSS* **(d)** *AAS*

Question 10.

If $A = \begin{bmatrix} 5 & 10 \\ 3 & -4 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then AI is equal to: [1]

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 5 & 10 \\ -3 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 10 \\ 3 & -4 \end{bmatrix}$ **(d)** $\begin{bmatrix} 15 & 15 \\ -1 & -1 \end{bmatrix}$

Question 11.

The polynomial $x^3 - 2x^2 + ax + 12$ when divided by $(x+1)$ leaves, remainder 20, then 'a' is equal to : [1]

(a) –31 **(b)** 9 **(c)** 11 **(d)** – 11

Question 12.

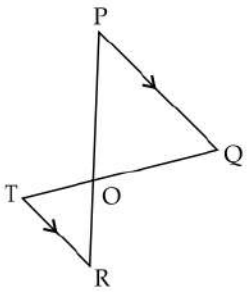
In an Arithmetic Progression (A.P.) if, first term is 5, common difference is – 3 and the n^{th} term is – 7, then n is equal to: [1]

(a) 5 **(b)** 17 **(c)** – 13 **(d)** 7

Question 13.

In the given figure PQ is parallel to TR, then by

using condition of similarity: [1]



(a) $\frac{PQ}{RT}=\frac{OP}{OT}=\frac{OQ}{OR}$ (b) $\frac{PQ}{RT}=\frac{OP}{OR}=\frac{OQ}{OT}$

(c) $\frac{PQ}{RT}=\frac{OR}{OP}=\frac{OQ}{OT}$ (d) $\frac{PQ}{RT}=\frac{OP}{OR}=\frac{OT}{OQ}$

Question 14.

If a, b, c, and d are proportional then $\frac{a+b}{a-b}$ is equal to: [1]

(a) $\frac{c}{d}$ (b) $\frac{c-d}{c+d}$

(c) $\frac{d}{c}$ (d) $\frac{c+d}{c-d}$

Question 15.

The first four terms of an Arithmetic Progression (A. P.), whose first term is 4 and common difference is -6, are: [1]

(a) $4, -10, -16, -22$ (b) 4, 10, 16, 22

(c) $4, -2, -8, -14$ (d) 4, 2, 8, 14

Question 16.

One of the roots of the quadratic equation $x^2 - 8x + 5 = 0$ is 7.3166. The root of the equation correct to 4 significant figures is: [1]

(a) 7.3166 (b) 7.317

(c) 7.316 (d) 7.32

Question 17.

$(x + 2)$ and $(x + 3)$ are two factors of the polynomial $x^3 + 6x^2 + 11x + 6$. If this polynomial is completely factorised the result is: [2]

(a) $(x-2)(x+3)(x+1)$

(b) $(x+2)(x-3)(x-1)$

(c) $(x+2)(x+3)(x-1)$

(d) $(x+2)(x+3)(x+1)$

Question 18.

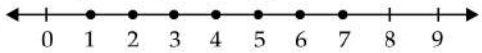
The sum of the first 20 terms of the Arithmetic Progression 2, 4, 6, 8, ... is: [2]

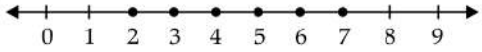
(a) 400 (b) 840 (c) 420 (d) 800

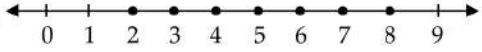
Question 19.

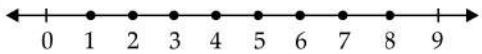
The solution set on the number line of the linear inequation: [2]

$2y - 6 < y + 2 \le 2y, y \in N$ is

(a)


(b)


(c)


(d)


Question 20.

If x, y, z are in continued proportion then $(y^2 + z^2) : (x^2 + y^2)$ is equal to: [2]

(a) $z : x$ (b) $x : z$

(c) zx (d) $(y + z) : (x + y)$

Question 21.

The marked price of an article is ₹ 5000. The shopkeeper gives a discount of 10%. If the rate of GST is 12%, then the amount paid by the customer including GST is: [2]

(a) ₹5040 (b) ₹6100 (c) ₹6272 (d) ₹6160

Question 22.

If $A = \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$, then $5A - BC$ is equal to: [2]

(a) $\begin{bmatrix} -5 & -23 \\ 1 & 17 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 23 \\ 1 & 17 \end{bmatrix}$

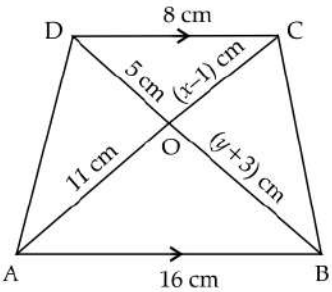
(c) $\begin{bmatrix} -2 & 8 \\ -3 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 23 \\ -1 & 17 \end{bmatrix}$

Question 23.

In the given figure $ABCD$ is a trapezium in which DC is parallel to AB.

$AB = 16$ cm and $DC = 8$ cm. $OD = 5$ cm, $OB = (y + 3)$ cm, $OA = 11$ cm and $OC = (x - 1)$ cm.

Using the given information answer the following questions.



(i) From the given figure name the pair of similar triangles: [1]

(a) $\Delta OAB, \Delta OBC$ (b) $\Delta COD, \Delta AOB$

(c) $\Delta ADB, \Delta ACB$ (d) $\Delta COD, \Delta COB$

(ii) The corresponding proportional sides with

respect to the pair of similar triangles obtained in (i): [1]

(a) $\frac{CD}{AB}=\frac{OC}{OA}=\frac{OD}{OB}$ (b) $\frac{AD}{BC}=\frac{OC}{OA}=\frac{OD}{OB}$

(c) $\frac{AD}{BC}=\frac{BD}{AC}=\frac{AB}{DC}$ (d) $\frac{OD}{OB}=\frac{CD}{CB}=\frac{OC}{OA}$

(iii) The ratio of the sides of the pair of similar triangles is: [1]

(a) 1 : 3 **(b)** 1 : 2

(c) 2 : 3 **(d)** 3 : 1

(iv) Using the ratio of sides of the pair of similar triangles the values of x and y are respectively: [1]

(a) $x = 4.6, y = 7$ **(b)** $x = 7, y = 7$

(c) $x = 6.5, y = 7$ **(d)** $x = 6.5, y = 2$

Question 24.

Two cars X and Y use 1 litre of diesel to travel x km and $(x + 3)$ km respectively. If both the cars covered a distance of 72 km, then:

(i) The number of litres of diesel used by car X is: [1]

(a) $\frac{72}{x-3}$ litres **(b)** $\frac{72}{x+3}$ litres

(c) $\frac{72}{x}$ litres **(d)** $\frac{12}{x}$ litres

(ii) The number of litres of diesel used by car Y is: [1]

(a) $\frac{72}{x-3}$ litres **(b)** $\frac{72}{x+3}$ litres

(c) $\frac{72}{x}$ litres **(d)** $\frac{12}{x+3}$ litres

(iii) If car X used 4 litres of diesel more than car Y in the journey, then [1]

(a) $\frac{72}{x-3}-\frac{12}{x}=4$ **(b)** $\frac{72}{x+3}-\frac{72}{x}=4$

(c) $\frac{72}{x}-\frac{72}{x+3}=4$ **(d)** $\frac{72}{x-3}-\frac{72}{x+3}=4$

(iv) The amount of diesel used by the car X is: [1]

(a) 6 litres **(b)** 12 litres

(c) 18 litres **(d)** 24 litres

Question 25.

Joseph has a recurring deposit account in a bank for two years at the rate of 8% per annum simple interest.

(i) If at the time of maturity Joseph receives ₹ 2000 as interest then the monthly installment is: [1]

(a) ₹ 1200 **(b)** ₹ 600

(c) ₹ 1000 **(d)** ₹ 1600

(ii) The total amount deposited in the bank: [1]

(a) ₹ 25000 **(b)** ₹ 24000

(c) ₹ 26000 **(d)** ₹ 23000

(iii) The amount Joseph receives on maturity is: [1]

(a) ₹ 27000 **(b)** ₹ 25000

(c) ₹ 26000 **(d)** ₹ 28000

(iv) If the monthly installment is ₹ 100 and the rate of interest is 8%, in how many months Joseph will receive ₹ 52 as interest? [1]

(a) 18 **(b)** 30

(c) 12 **(d)** 6

SOLUTIONS

1. Option (b) is correct.

Explanation:

$$x + 2 = 0$$
$$x = -2$$
$$p(x) = x^3 - kx^2 - 5x + 6 = 0$$

Remainder $= p(-2) = 0$

$$(-2)^3 - k(-2)^2 - 5(-2) + 6 = 0$$
$$-8 - 4k + 10 + 6 = 0$$
$$-4k = -8$$
$$k = 2$$

2. Option (c) is correct.

Explanation:

$$x - 3 \geq -5$$
$$x \geq -5 + 3$$
$$x \geq -2$$

3. Option (b) is correct.

Explanation: The product of AB of two matrices A and B is possible if the number of column in A is equal to the number of rows in B.

4. Option (c) is correct.

Explanation: $a = 70, d = 75 - 70 = 5$

$$a_n = a + (n-1)d$$
$$a_{10} = 70 + (10-1)5$$
$$= 70 + 45$$
$$a_{10} = 115$$

5. Option (c) is correct.

Explanation: $SP = ₹\ 800, GST = 12\%$

$$GST = ₹\ \frac{12}{100} \times 800 = ₹\ 96$$

$$\text{Total Cost} = 800 + 96 = ₹\ 896$$

6. Option (a) is correct.

Explanation:

$\because$ $x^2 - (\text{sum of roots})x - (\text{product of roots}) = 0$

$\therefore$ $x^2 - (2 + 3)x - (2 \times 3) = 0$

$\Rightarrow$ $x^2 - 5x + 6 = 0$

7. Option (a) is correct.

Explanation: Numbers are in proportion (given)

$$\Rightarrow \quad \frac{x}{5.4} = \frac{5}{9}$$

$$\Rightarrow \quad 9 \times x = 5.4 \times 5$$

$$\Rightarrow \quad x = \frac{5.4 \times 5}{9} = 0.6 \times 5$$

$$\Rightarrow \quad x = 3$$

8. Option (d) is correct.

Explanation: Maturity Amount = ₹25500

Total deposit = 1000 × 24 = ₹24000

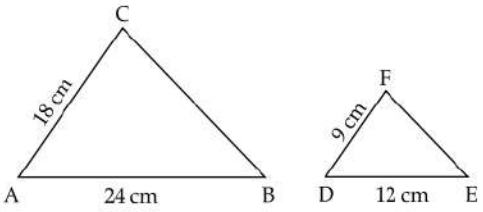
$\therefore$ Interest = ₹25,500 – ₹24,000

= ₹1,500

9. Option (b) is correct.

Explanation: In ΔDEF & ΔABC,

$$\frac{AC}{DF} = \frac{AB}{DE} = \frac{2}{1}$$



$\angle BAC = \angle EDF$

Hence, $\Delta ABC \sim \Delta DEF$ (by *SAS* criterion)

10. Option (c) is correct.

Explanation: $A \times I = A$

11. Option (d) is correct.

Explanation: $x + 1 = 0$

$x = -1$

Remainder = $p(-1)$

Let $p(x) = x^3 - 2x^2 + ax + 12$

Now, $p(-1) = 20$

$\Rightarrow \quad (-1)^3 - 2(-1)^2 + a(-1) + 12 = 20$

$\Rightarrow \quad -1 - 2 - a + 12 = 20$

$\Rightarrow \quad -a = 20 - 9$

$\Rightarrow \quad a = -11$

12. Option (a) is correct.

Explanation: Given, $a = 5, d = -3$ and $a_n = -7$

Since, $a_n = a + (n-1)d$

$\therefore \quad -7 = 5 + (n-1)(-3)$

$\Rightarrow \quad -7 - 5 = -3n + 3$

$\Rightarrow \quad -12 - 3 = -3n$

$\Rightarrow \quad -15 = -3n$

$\Rightarrow \quad n = 5$

13. Option (b) is correct.

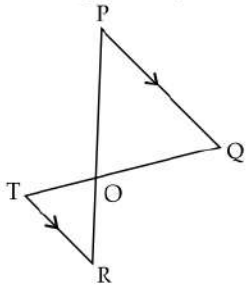
Explanation:

$PQ \parallel TR$, PR is transversal,

$\Rightarrow \quad \angle P = \angle R$ (alternate angles)

Also, QT is transversal,

$\Rightarrow \quad \angle Q = \angle T$ (alternate angles)



$\therefore \quad \Delta RTO \sim \Delta PQO$ (by AA criterion)

$$\therefore \frac{PQ}{RT} = \frac{PO}{RO} = \frac{QO}{TO} \quad \text{(by C.P.C.T)}$$

14. Option (d) is correct.

Explanation:

Given, $\frac{a}{b} = \frac{c}{d}$

By compodendo-dividendo

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

15. Option (c) is correct.

Explanation: $a = 4$; $d = -6$

First four terms of an A.P. are:

$a, a + d, a + 2d, a + 3d$

$\Rightarrow \quad 4, 4 + (-6), 4 + 2(-6), 4 + 3(-6)$

$\Rightarrow \quad 4, -2, -8, -14$

16. Option (b) is correct.

Explanation: Correct to 4 significant figures is 7.317.

17. Option (d) is correct.

Explanation: Let $p(x) = x^3 + 6x^2 + 11x + 6$

If $x + 1 = 0$

then $x = -1$

Now, $p(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$

$= -1 + 6 - 11 + 6 = 0$

Hence, $(x + 1)$ is a third factor.

18. Option (c) is correct.

Explanation: $S_n = \frac{n}{2}[2a + (n-1)d]$

Here, $a = 2, d = 4 - 2 = 2$ and $n = 20$

$\therefore \quad S_{20} = \frac{20}{2}[2 \times 2 + (20-1)2]$

$= 10[4 + 38]$

$= 10 \times 42 = 420$

19. Option (b) is correct.

Explanation: $2y - 6 < y + 2$

$2y - y < 2 + 6$

$y < 8$

and, $y + 2 \leq 2y$

$2 \leq 2y - y$

$2 \leq y$

So, $2 \leq y < 8$

20. Option (a) is correct.

Explanation: x, y, z are in proportion

$$y^2 = xz$$

$$\therefore \quad \frac{y^2 + z^2}{x^2 + y^2} = \frac{xz + z^2}{x^2 + xz} = \frac{z(x+z)}{x(x+z)} = \frac{z}{x}$$

i.e., $z : x$

21. Option (a) is correct.

Explanation: $SP = MP - \text{Discount}$

$$= 5000 - \frac{10}{100} \times 5000$$

$$= 5000 - 500 = 4500$$

$GST = 12\%$

$$GST \text{ amount} = \frac{12}{100} \times 4500 = 540$$

Total amount paid = 4500 + 540 = ₹5040

22. Option (d) is correct.

Explanation:

$$5A - BC = 5\begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 25 \\ 5 & 20 \end{bmatrix} - \begin{bmatrix} 10 & 2 \\ 6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 23 \\ -1 & 17 \end{bmatrix}$$

23. (i) Option (b) is correct.

Explanation: $DC \| AB$ (given)

$\angle CDO = \angle ABO$ (alternate angle)

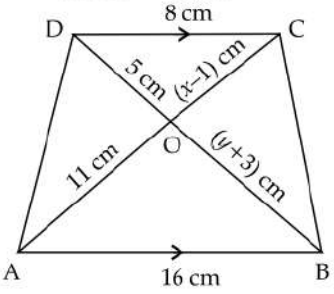
$\angle DCO = \angle BAO$ (alternate angle)

$\therefore \quad \Delta COD \sim \Delta AOB$ (by *AA* criterion)

(ii) Option (a) is correct.

Explanation:

$\Delta COD \sim \Delta AOB$



$$\therefore \quad \frac{CD}{AB} = \frac{OD}{OB} = \frac{CO}{AO} \qquad \text{(by C.P.C.T)}$$

(iii) Option (b) is correct.

Explanation:

$$\frac{CD}{AB} = \frac{OD}{OB} = \frac{CO}{AO} = \frac{8}{16} = \frac{1}{2}$$

(iv) Option (c) is correct.

Explanation:

Here, $\frac{x-1}{11} = \frac{5}{y+3} = \frac{1}{2}$

$\therefore \frac{x-1}{11} = \frac{1}{2}$ and $\frac{5}{y+3} = \frac{1}{2}$

$\Rightarrow 2x - 2 = 11$ and $y + 3 = 10$

$\Rightarrow x = \frac{13}{2} = 6.5$ and $y = 7$

24. (i) Option (c) is correct.

Explanation: x km = 1 litre

$$72 \text{ km} = \frac{72}{x} \text{ litre}$$

(ii) Option (b) is correct.

Explanation: $(x + 3)$km = 1 litre

$$72 \text{ km} = \frac{72}{x+3} \text{ litre}$$

(iii) Option (c) is correct.

Explanation: Since, given car X used 4 litres of diesel more than car Y.

Therefore, $\frac{72}{x} - \frac{72}{x+3} = 4$

(iv) Option (b) is correct.

Explanation: We have, $\frac{72}{x} - \frac{72}{x+3} = 4$

$\Rightarrow \quad 72(x + 3) - 72x = 4x^2 + 12x$

$\Rightarrow \quad 72x + 216 - 72x = 4x^2 + 12x$

$\Rightarrow \quad 4x^2 + 12x - 216 = 0$

$\Rightarrow \quad x^2 + 3x - 54 = 0$

$\Rightarrow \quad x^2 + 9x - 6x - 54 = 0$

$\Rightarrow \quad x(x + 9) - 6(x + 9) = 0$

$\Rightarrow \quad (x - 6)(x + 9) = 0$

$\Rightarrow \quad x = 6$ (distance cannot be –ve)

$\therefore$ Amount of diesel used by car X is $\frac{72}{6} = 12$ litres

25. (i) Option (c) is correct.

Explanation: Given, $R = 8\%$; $I = ₹\ 2000$, $n = 24$

$$I = P \times \frac{n(n+1)}{2} \times \frac{R}{1200}$$

$$2000 = P \times \frac{24 \times 25}{2} \times \frac{8}{1200}$$

$$2000 = P \times 12 \times \frac{8}{48}$$

$$P = \frac{2000}{2} = ₹1000$$

(ii) Option (b) is correct.

Explanation: Total amount = ₹ 1000 × 24
= ₹ 24,000

(iii) Option (c) is correct.

Explanation: Maturity amount
= Total amount + Interest
= ₹ 24000 + ₹ 2000
= ₹ 26000

(iv) Option (c) is correct.

Explanation: $R = 8\%$; $I = ₹ 52$, $P = ₹100$

$$I = P \times \frac{n(n+1)}{2} \times \frac{R}{1200}$$

$$52 = 100 \times \frac{n(n+1)}{2} \times \frac{8}{1200}$$

$$n(n+1) = 3 \times 52$$
$$n^2 + n - 156 = 0$$
$$n^2 + 13n - 12n - 156 = 0$$
$$n(n+13) - 12(n+13) = 0$$
$$(n+13)(n-12) = 0$$
$$\therefore \quad n = 12 \text{ months}$$

❑❑❑

ICSE SEMESTER-2 EXAMINATION

Mathematics

Solved Paper - 2022

Class-10th

Maximum Marks: 40
Time Allowed: One and a half hours
Answer to this Paper must be written on the paper provided separately.
You will not be allowed to write during the first 10 minutes.
This time is to be spent in reading the question paper.
The time given at the head of this Paper is the time allowed for writing the answers.

Attempt all questions from Section A and any three questions from Section B.
The marks intended for questions are given in brackets [ ]
Mathematical tables are provided.

SECTION A

*(Attempt **all** questions.)*

1. Choose the correct answers to the questions from the given options. (Do not copy the question. Write the correct answer only.) [10]

(i) The probability of getting a number divisible by 3 in throwing a dice is:

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(ii) The volume of a conical tent is 462 m^3 and the area of the base is 154 m^2. The height of the cone is:

(a) 15 m (b) 12 m
(c) 9 m (d) 24 m

(iii) The median class for the given distribution is:

Class Interval	0 - 10	10 - 20	20 - 30	30 - 40
Frequency	2	4	3	5

(a) 0-10 (b) 10-20
(c) 20-30 (d) 30-40

(iv) If two lines are perpendicular to one another then the relation between their slopes m_1 and m_2 is:

(a) $m_1 = m_2$ (b) $m_1 = \frac{1}{m_2}$

(c) $m_1 = -m_2$ (d) $m_1 \times m_2 = -1$

(v) A lighthouse is 80 m high. The angle of elevation of its top from a point 80 m away from its foot along the same horizontal line is:

(a) 60° (b) 45°
(e) 30° (d) 90°

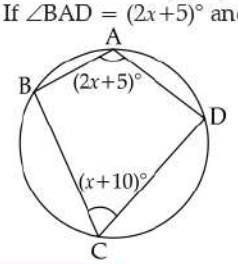
(vi) "The model class of a given distribution always corresponds to the:

(a) interval with highest frequency (b) interval with lowest frequency
(c) the first interval (d) the last interval

(vii) The coordinates of the point P(-3,5) on reflecting on the X axis are:

(a) (3, 5) (b) (–3, –5)
(c) (3. –5) (d) (–3, 5)

(viii) ABCD is a cyclic quadrilateral. If $\angle BAD = (2x+5)°$ and $\angle BCD = (x + 10)°$ then x is equal to:



(a) 65° (b) 45°
(c) 55° (d) 5°

(ix) A(1, 4), B (4, 1) and C (x, 4) are the vertices of ΔABC. If the centroid of the triangle is G (4, 3) then x is equal to
(a) 2 (b) 1
(c) 7 (d) 4

(x) The radius of a roller 100 cm long is 14 cm. The curved surface area of the roller is:
(Take $\pi = \frac{22}{7}$)
(a) 13200 cm^2 (b) 15400 cm^2
(c) 4400 cm^2 (d) 8800 cm^2

SECTION B

*(Attempt **any three** questions from this Section.)*

2.

(i) Prove that **[2]**

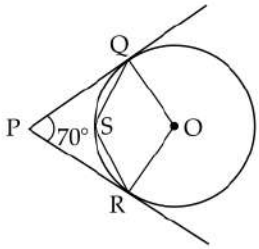
$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta} = 2\sec^2\theta$$

(ii) Find 'a'. if A (2a + 2, 3), B (7, 4) and C (2a + 5, 2) are collinear. **[2]**

(iii) Calculate the mean of the following frequency distribution **[3]**

Class Interval	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55
Frequency	2	6	4	8	4

(iv) In the given figure O is the centre of the circle. PQ and PR are tangents and ∠QPR = 70° Calculate. **[3]**

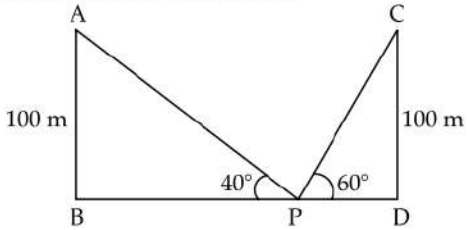


(a) ∠QOR (b) ∠QSR

3. (i) A bag contains 5 white, 2 red and 3 black balls. A ball is drawn at random. What is the probability that the ball drawn is a red ball? **[2]**

(ii) A solid cone of radius 5 cm and height 9 cm is melted and made into small cylinders of radius of 0.5 cm and height 1.5 cm. Find the number of cylinders so formed. **[2]**

(iii) Two lamp posts AB and CD each of height 100 m are on either side of the road. P is a point on the road between the two lamp posts. The angles of elevation of the top of the lamp posts from the point P are 60°and 40°. Find the distances PB and PD. **[3]**



(iv) Marks obtained by 100 students in an examination are given below. **[3]**

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
No of students	5	15	20	28	20	12

Draw a histogram for the given data using a graph paper and find the mode.
Take 2 cm = 10 marks along one axis and 2 cm = 10 students along the other axis.

4. (i) Find a point P which divides internally the line segment joining the points A (–3, 9) and B (1, –3) in the ratio 1 : 3. **[2]**

(ii) A letter of the word 'SECONDARY' is selected at random. What is the probability that the letter selected is not a vowel? **[2]**

(iii) Use a graph paper for this question. Take 2cm - 1 unit along both the axes. **[3]**

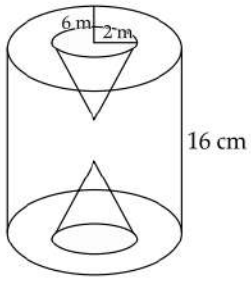
(a) Plot the points A (0, 4), B (2, 2), C(5, 2) and D (4, 0) E(0, 0) is the origin.

(b) Reflect B, C, D on the Y-axis and name them as B', C' and D' respectively.

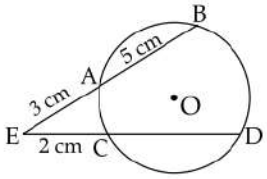
(c) Join the points ABCDD'C'B' and A in order and give a geometrical name to the closed figure.

(iv) A solid wooden cylinder is of radius 6 cm and height 16 cm. Two cones each of radius 2 cm and height 6 cm are drilled out of the cylinder. Find the volume of the remaining Solid. **[3]**

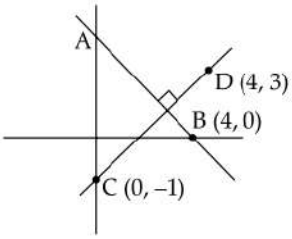
(Take $\pi = \frac{22}{7}$)



5. (i) Two chords AB and CD of a circle intersect externally at E, If EC = 2 cm, EA = 3cm and AB = 5cm, find the length of CD. **[2]**



(ii) Line AB is perpendicular to CD Coordinates of B, C and D respectively (4, 0), (0, –1) and (4, 3). **[2]**



Find

(a) Slope of CD (b) Equation of AB

(iii) Prove that: **[3]**

$$\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta} = \sec^2\theta + \tan^2\theta$$

(iv) The mean of the following distribution is 50 Find the unknown frequency **[3]**

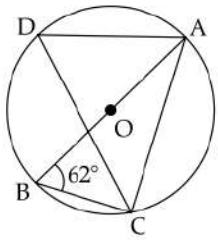
Class Interval	Frequency
0 - 20	6
20 - 40	f
40 - 60	8
60 - 80	12
80 - 100	8

6. (i) Prove that: **[2]**

$$1+\frac{\tan^2\theta}{1+\sec\theta} = \sec\theta$$

(ii) In the given figure A, B, C and D are points on the circle with centre O. Given $\angle ABC = 62°$. **[2]**

Find:



(a) $\angle ADC$ (b) $\angle CAB$

(iii) Find the equation of a line parallel to the line $2x + y - 7 = 0$ and passing through the intersection of the lines $x + y - 4 = 0$ and $2x - y = 8$. **[3]**

(iv) Marks obtained by 40 students in an examination are given below. **[3]**

Marks	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No of students	3	8	14	9	4	2

Using graph paper draw an ogive and estimate the median marks. Take 2 cm = 10 marks along one axis and 2 cm = 5 students along the other axis.

ANSWERS

1. (i) (b) $\frac{1}{3}$

Explanation: All possible out comes of a dice = {1, 2, 3, 4, 5, 6}

n (E) = 6

Favourable outcomes (divisible by 3) = {3, 6}

n (F) = 2

$$\text{Probability of getting a number divisible by 3} = \frac{\text{Favourable outcomes}}{\text{Total no. of outcomes}}$$

$$= \frac{n(F)}{n(E)} = \frac{2}{6} = \frac{1}{3}$$

(ii) (c) 9 m

Explanation: Given– Volume of conical tent = 462 m^3

and Area of the base = 154 m^2

We know that

$$\text{Volume of a conical tent} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \text{Area of base} \times \text{height}$$

$$\Rightarrow \quad 462 = \frac{1}{3} \times 154 \times h$$

$$\Rightarrow \quad h = \frac{462 \times 3}{154} = 9 \text{ m}$$

(iii) (c) 20 – 30

Explanation:

C. I.	Frequency	C. F.
0 - 10	2	2
10 – 20	4	2 + 4 = 6 (C. F.)
20 – 30	3 (f)	6 + 3 = 9
30 – 40	5	9 + 5 = 14
	14	

$$N = 14 \Rightarrow \frac{N}{2} = 7$$

We can see that cumulative frequency just greater than 7 is 9, which is lie in the class interval 20 – 30. So, the median class is 20 – 30.

(iv) (d) $m_1 \times m_2 = -1$

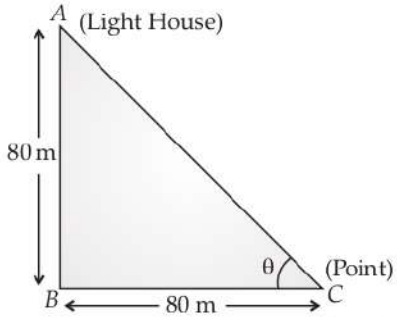
Explanation: If two lines, having slopes m_1 and m_2, perpendicular to one another, then $m_1 \times m_2 = -1$

(v) (b) 45°

Explanation: Height of the light house = 80 m

Distance of the point from the foot of the light house = 80 m

Let the angle of elevation be θ, then $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$



$$\tan\theta = \frac{AB}{BC} = \frac{80}{80} = 1$$

$$\tan\theta = \tan 45° \quad (\because \tan 45° = 1)$$

So, $\theta = 45°$

(vi) (a) interval with highest frequency.

(vii) (b) (–3, –5)

Explanation: Given point (–3, 5) lies in IInd quadrant. So, after reflection sign of Y-coordinate will invert and sign of X-coordinate remain same. Reflection of this point will be in IIIrd-quadrant, *i.e.*, (–3, –5)

(viii) (c) 55°

Explanation: Given– $\angle BAD = (2x + 5)°$

and $\angle BCD = (x + 10)°$

So, $\angle BAD + \angle BCD = 180°$

(Opposite angles of a cyclic quadrilateral are supplementary)

$\Rightarrow$ $(2x + 5)° + (x + 10)° = 180°$

$(3x + 15)° = 180°$

$3x = 165°$

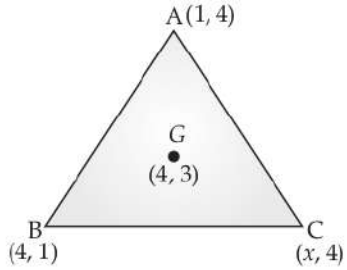
$x = \frac{165°}{3} = 55°$

(ix) (c) 7

Explanation: Coordinates of centroid of a triangle is given by

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

So, coordinates of the centroid of triangle ABC is given by $\left(\frac{1+4+x}{3}, \frac{4+1+4}{3}\right)$ or $\left(\frac{5+x}{3}, 3\right)$



Also, Coordinates of centroid of ΔABC (4, 3)

So, $4 = \frac{5+x}{3} \Rightarrow 12 = 5 + x \Rightarrow x = 7$

(x) (d) 8800 cm^2

Explanation: We have,

radius of a roller = 14 cm

height of the roller = 100 cm

Now, C.S.A of roller (Cylinder) = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 14 \times 100 = 8800 \text{ cm}^2$$

SECTION B

2. (i) L.H.S,

$$= \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$$

$$= \frac{(1-\sin\theta)+(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta} \qquad (\because \sin^2\theta + \cos^2\theta = 1)$$

$= 2\sec^2\theta$ = R.H.S. Hence Proved. $\left(\because \frac{1}{\cos\theta} = \sec\theta\right)$

(ii) Given A $(2a + 2, 3)$, B$(7, 4)$ and C $(2a + 5, 2)$ are collinear.

When three points are colliner, then $\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right] = 0$

or $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

We have

$x_1 = 2a + 2, \quad x_2 = 7, \quad x_3 = 2a + 5$

$y_1 = 3, \quad y_2 = 4, \quad y_3 = 2$

$\Rightarrow \quad (2a + 2)(4 - 2) + 7(2 - 3) + (2a + 5)(3 - 4) = 0$

$4a + 4 - 7 - 2a - 5 = 0$

$2a - 8 = 0$

$a = \frac{8}{2} = 4$

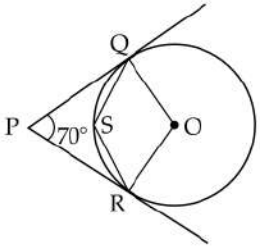
(iii)

C. I.	Class mark (x)	Frequency (f)	$f \times x$
5 – 15	10	2	20
15 – 35	20	6	120
25 – 35	30	4	120
35 – 45	40	8	320
45 – 55	50	4	200
Total		24	780

We know, $\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}$

$$\bar{x} = \frac{780}{24} = 32.5$$

(iv) PQOR is a quadrilateral.



(a) $\angle QPR + \angle QOR = 180°$ (supplementary angles)

$70° + \angle QOR = 180°$

$\angle QOR = 110°$

(b) Reflex $\angle QOR = 360° - 110° = 250°$

$\angle QSR = \frac{1}{2}$ Reflex $\angle QOR$

($\because$ The angle subtended by an arc at the centre is double the angle subtended by it on the remaining part of the circle.)

$$\angle QSR = \frac{1}{2} \times 250° = 125°$$

3. (i) Given, No. of White balls = 5
No. of red balls = 2
No. of black balls = 3
Total no. of balls = 5 + 2 +3 = 10

Probability of getting a red ball

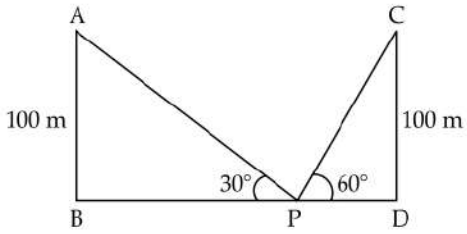
$$= \frac{\text{No. of favourable outcomes (red balls)}}{\text{Total no. of balls}} = \frac{2}{10} = \frac{1}{5}$$

(ii) Given,
For cone,
Radius (R) = 5 cm, Height (H) = 9 cm
For cylinders,
radius (r) = 0.5 cm, height (h) = 1.5 cm

$$\text{No. of cylinders} = \frac{\text{Volume of Cone}}{\text{Volume of cylinder}}$$

$$= \frac{\frac{1}{3}\pi R^2 H}{\pi r^2 h} = \frac{\frac{1}{3}\times(5)^2\times 9}{(0.5)^2\times 1.5} = \frac{25\times 3}{0.25\times 1.5} = 200$$

(iii) Given,
Heights of the lamp posts AB = CD = 100 m
and $\angle APB = 30°$; $\angle CPD = 60°$
Now, In ΔAPR



$$\tan P = \frac{\text{Perpendicular(AB)}}{\text{Base}}$$

$$\tan 30° = \frac{AB}{BP} = \frac{100}{BP}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{BP} \Rightarrow BP = 100\sqrt{3}\text{ m}$$

$$BP = 100 = 1732 = 173.2\text{ m}$$

Correction–In this question paper $\angle APB = 40°$ is given, which should be 30° to solve it.
Now, In ΔPCB

$$\tan P = \frac{CD}{PD} = \frac{100}{PD}$$

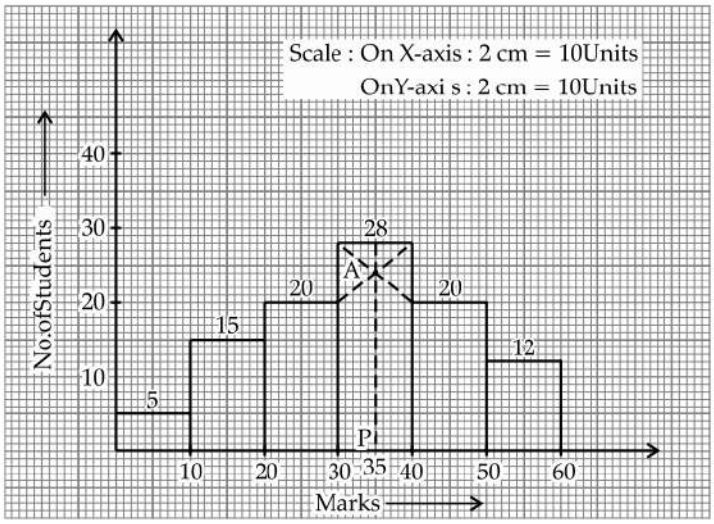
$$\tan 60° = \frac{100}{PD}$$

$$\sqrt{3} = \frac{100}{PD}$$

$$\Rightarrow \quad PD = \frac{100}{\sqrt{3}}\text{ m} = \frac{100\sqrt{3}}{3} = \frac{173.2}{3}$$

$$PD = 57.73\text{ m}$$

(iv) Mode = 35



To find mode.

(i) first identify the rectangle with highest frequency (Modal class); here 30–40.

(ii) Join the top corners of the modal rectangle with immediate next corners of the adjacent rectangles.

(iii) Let the point where the joining lines cut each other (here A). Draw a perpendicular from A to X-axis. The point 'P', where the perpendicular meet the X-axis will give the mode.

4. (i) Given points are

$A(x_1, y_1) = (-3, 9)$

and $B(x_2, y_2) = (1, -3)$

Let P(x, y) divides the line segment AB in the ratio 1:3. Then by using section formula,

$$= \left(\frac{1\times1+3\times-3}{1+3}, \frac{1\times-3+3\times9}{1+3}\right)$$

$$P(x, y) = \left(\frac{-8}{4}, \frac{24}{4}\right)$$

$$P(x, y) = (-2, 6)$$

(ii) Given word is 'SECONDARY'

No. of vowels = 3

No. of Consonants = 6

Total no. of letters = 9

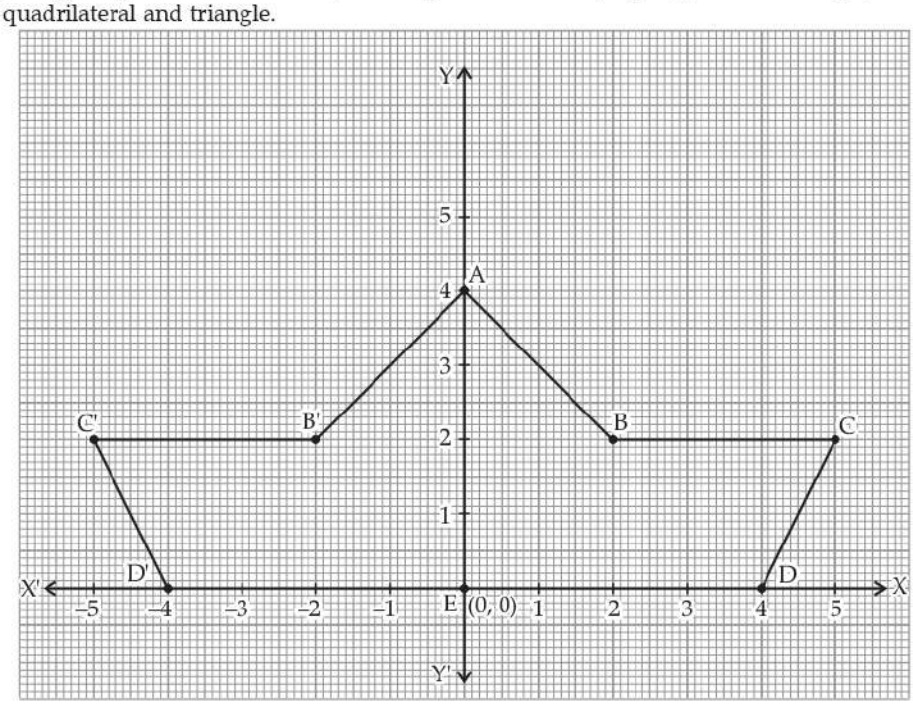
Probability of not selecting a vowel = Probability of selecting a consonant

$$P \text{ (not a vowel)} = \frac{6}{9} = \frac{2}{3}$$

(iii) (a) See graph

(b) See graph

(c) On joining the points ABCDD'C'B'A, the shape obtain is not proper geometrical shape, but collection of a quadrilateral and triangle.



(iv) Given:

for Cylinder,

Radius (R) = 6 cm

Height (H) = 16 cm

We know, Volume = $\pi R^2 H$

$$= \frac{22}{7} \times (6)^2 \times (16)$$

$$= \frac{12672}{7}\text{cm}^3$$

for cone,

radius (r) = 2 cm

height (h) = 6 cm

We know, volume $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (2)^2 \times 6$$

$$= \frac{176}{7}\text{cm}^3$$

Remaining Volume = Volume of cylinder – 2 × Volume of a cone

$$= \frac{12672}{7} - 2 \times \frac{176}{7} = \frac{12672}{7} - \frac{352}{7}$$

$$= \frac{12672 - 352}{7}$$

$$= \frac{12320}{7}$$

$$= 1760 \text{ cm}^3$$

5. (i) From the given figure,

We have EA = 3 cm, AB = 5 cm

EC = 2 cm

When two chords intersect externally, then (from figure)

$$EA \times EB = EC \times ED$$

or $EA \times (EA + AB) = EC \times (EC + CD)$

$\Rightarrow$ $3 \times (3 + 5) = 2\,(2 + CD)$

$$24 = 4 + 2CD$$

$\Rightarrow$ $$2CD = 20$$

$$CD = 10 \text{ cm}$$

$$x + y - 4 = 0$$

(ii) (a) Slope of line CD is given by

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{0-4} = 1$$

(b) Slope of line AB, Which is perpendicular to CD given by

$$m = \frac{1}{-m} = -\frac{1}{1} = -1$$

Now, equation of a line (AB), when slope and a coordinates is given

$$y - y_1 = m_1 (x - x_1)$$

where $y_1 = 0, m_1 = -1, x_1 = 4$

So, $y - 0 = -1\,(x - 4)$

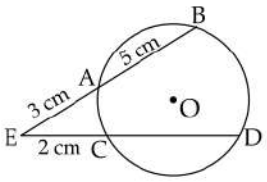
$$y = -x + 4$$

(iii) L.H.S

$$= \frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{2\cos^2\theta}$$

$$= \frac{1^2 + \sin^2\theta + 2\sin\theta + 1^2 + \sin^2\theta - 2\sin\theta}{2\cos^2\theta}$$

$$= \frac{2 + 2\sin^2\theta}{2\cos^2\theta} = \frac{1 + \sin^2\theta}{\cos^2\theta}$$



$$= \frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$$

$= \sec^2\theta + \tan^2\theta$ = R.H.S. **Hence Proved.**

(iv)

C. I.	Class mark (x)	Frequency (f)	$f \times x$
0 – 20	10	6	60
20 – 40	30	f	$30f$
40 – 60	50	8	400
60 – 80	70	12	840
80 – 100	90	8	720
Total		$34 + f$	$2020 + 30f$

We know, $\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}$

$$50 = \frac{2020 + 30f}{34 + f} \quad \text{(given)}$$

$$1700 + 50f = 2020 + 30f$$
$$50f - 30f = 2020 - 1700$$
$$20f = 320$$
$$f = 16$$

6. (i) L.H.S.

$$= 1 + \frac{\tan^2\theta}{1+\sec\theta}$$

$$= 1 + \frac{\left(\sec^2\theta - 1\right)}{1+\sec\theta} \quad (\because \sec^2\theta - \tan^2\theta = 1)$$

$$= 1 + \frac{(\sec\theta + 1)(\sec\theta - 1)}{(1+\sec\theta)}$$

$= 1 + \sec\theta - 1$

$= \sec\theta$ = R.H.S.

(ii) (a) Given, $\angle ABC = 62°$

$\angle ABC = \angle ADC = 62°$ ($\because$ Angles in the same segment are equal)

(b) In ΔABC,

$\angle ABC = 62°$

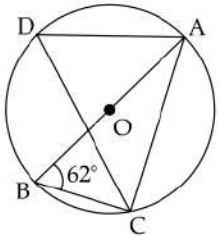
$\angle ACB = 90°$ ($\because$ Angles in a semicircle is a right angle)

Now, $\angle ABC + \angle ACB + \angle CAB = 180°$ (Angle sum property)

$62° + 90° + \angle CAB = 180°$

$\angle CAB = 180° - 152°$

$\angle CAB = 28°$



(iii) Given,

$$x + y - 4 = 0 \text{ or } x + y = 4$$

and $2x - y = 8$

On solving above two equations we get the intersection points as (4, 0)

Now, equation of line parallel to $2x + y - 7 = 0$ is given by

$$2x + y = \lambda$$

which is passing through (4, 0)

So, $2 \times 4 + 0 = \lambda$

$$\lambda = 8$$

So, the required equation of the line is $2x + y = 8$.

$$2x + y - 8 = 0$$

(iv)

Marks	No. of students	Marks less than	Cumulative frequency
10 – 20	3	20	3
20 – 30	8	30	11
30 – 40	14	40	25
40 – 50	9	50	34
50 – 60	4	60	38
60 – 70	2	70	40

Other than the given class intervals, we assume a class interval 0 – 10 prior the first class with zero frequency.

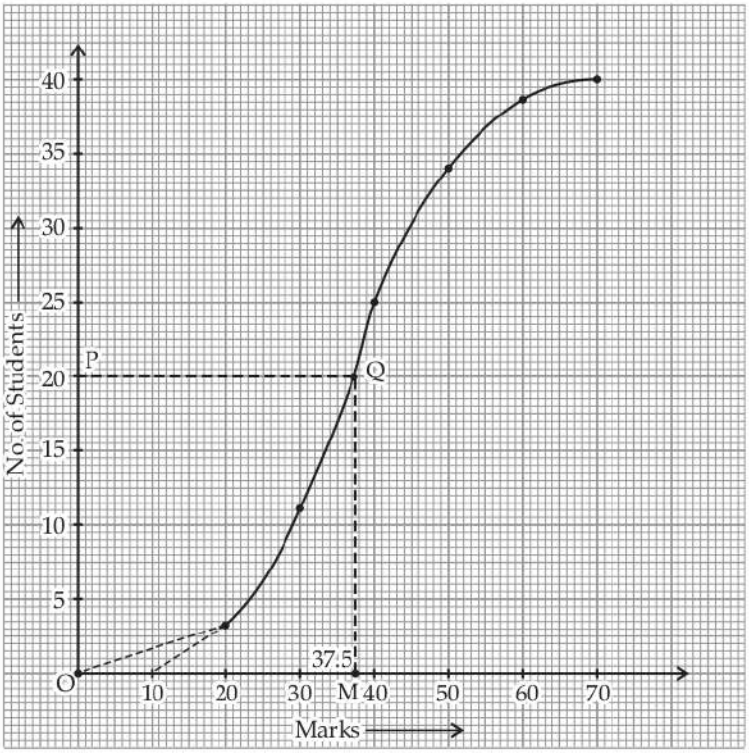
Now, plot the points (10, 0), (20, 3), (30, 11), (40, 25), (50, 34), (60, 38), (70, 40) on the graph paper.

In order to obtain ogive, we draw a smooth curve passing through these points.

In order to find the median, we first locate the point $\frac{N}{2} = \frac{40}{2} = 20$ on Y-axis. Let the point P from this draw a line parallel to x-axis cutting the curve at Q from Q draw a line parallel to y-axis meeting the x-axis at point M. The coordinate of M is 37.5. Hence, the median is 37.5.

Now plot the points (10, 0), (20, 3), (30, 11), (40, 25), (50, 34), (60, 38), (70, 40) on the graph paper.

In order to obtain ogive, we draw a smooth curve passing through these, points.



■■